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ABSTRACT

This manual provides a relevant resource for mathematics teachers who wish to take advantage of the extensive library of computer software materials available to expand and strengthen classroom instruction. Classroom management procedures, software duplication guidelines, procedures for copying diskettes for MS DOS and Apple II computers, and methods for maintaining diskettes are described. Included are 59 classroom activities for teaching mathematical concepts using a computer. Included for each of the activities are: (1) the aim; (2) objectives; (3) vocabulary; (4) materials; (5) preparation; (6) motivation; (7) development; (8) application; (9) summary; and (10) homework. A list of 37 software items used in this guide is presented. (YP)

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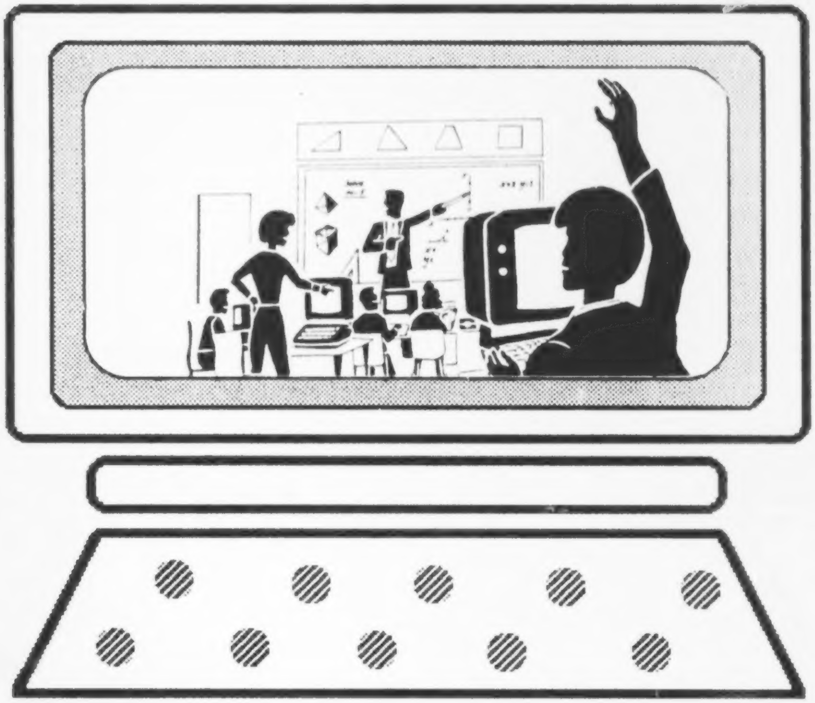

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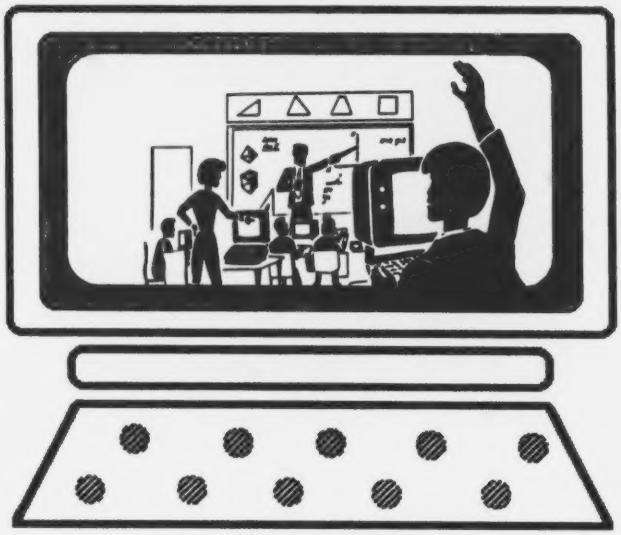

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Foreword

The use of computers in the classroom has become more prevalent during the past few years. Through this use we have developed a knowledge of their functions and a better vision of their potential. We encourage our colleagues to freely experiment and attempt new approaches. Only with all of us working in concert, can we fulfill our vision for an effective integration of school technology into all subject areas. From these beginnings, we will nurture the critical thinking and practical skills our students will need to succeed in the information age. If children are not comfortable with computers, they will be as unprepared for the future as an adult who cannot read or write.

The purpose of this guide is to give teachers and supervisors a working knowledge of various approaches to enhance pupil learning. Our guides are intended to be open ended and flexible in nature. As a 1920 car is not efficient by 1980 standards, outmoded computer education ideas of the seventies have been replaced by the new visions of the eighties. Classroom use of technology has doubled and redoubled in the past few years. Teachers are continually devising new ideas for the use of this technology.

Feel free to write to me directly with your comments and suggestions concerning this curriculum guide, so that we may improve and update it to meet your needs.

Irwin Kaufman

Irwin Kaufman, Chief Information Officer
New York City Board of Education

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Introduction: How to Use This Guide

The purpose of this manual is to provide a relevant resource for mathematics teachers who wish to take advantage of the extensive library of computer software materials available to expand and strengthen classroom instruction, as well as use alternative methodology.

Sample Computer Applications for the High School Mathematics Classroom presents selected lesson plans which will facilitate the integration of computer-based instruction with the regular methods practiced by teachers. These plans not only indicate the features of the software, but also represent a sharing of ideas on its applications by teachers who have had successful experiences with computers and enriching mathematical concepts and skills.

It must be emphasized that this collection of suggested lessons is not intended as a sequential course, but rather as a reference and guide from which teachers may draw to the extent and in the manner they deem appropriate for their students.

Teachers should familiarize themselves with the section of this book on "Classroom Management Procedures" before using the computer lab with their classes. Those who require more information on specific programs, such as electronic spreadsheets or data bases, are referred to the Division of Computer Information Services publications Apple Tool Software and MS DOS Tool Software. Further assistance may always be obtained from your local Technical Assistance Center. (See "For More Help...")

It is expected that as teachers gain in experience with the computer as a teaching tool and in familiarity with the software content and flexibility of its use, they will not only utilize the plans increasingly, but will also devise many new strategies for future development of these materials.

PREFACE: Computers — Teaching Tools

Computers were obviously not in mind when antislavery preacher Henry Ward Beecher offered this defense of technology over a hundred years ago:

> "A tool is but an extension of a man's hand, and
> a machine is but a complex tool..."

We may nevertheless find in it encouragement in considering their applicability to teaching and learning. Teachers already seasoned in computer-based instruction understand its power in raising instructional effectiveness, and are free of unwarranted concern about bypassing human reason and the loss of affective considerations. Others, at the threshold of engaging this seemingly robotic methodology, may confidently expect that even these complex machines are simply extended tools of instruction. They are fully controlled by the instructor's hands as readily as the conventional ones - chalk, textbooks and the like.

Teachers may anticipate at least the following advantages to incorporating the computer into their regular instructional program:

. students may study individually, and yet together, on synchronized exercises

. each student is required to focus attention narrowly; the graphic, repeatable presentations and explanations may ensure a clearer understanding of the lesson's topic

. it allows a greater immediacy of active, individual response for a greater number of students

. it gives students immediate feedback to their responses

. students may work at their own pace rather than at the pace of an imaginary class median

. it makes possible a more extended contact with the subject matter, as an individual or as part of a small group

. it affords absentees a certain and convenient way to catch up on missed studies

. it opens up to teachers strategies to make more efficient use of class time than some traditional question-and-answer lessons

. students' responses are treated in a nonjudgemental way, which may make learning more comfortable for some students; it never gets tired of a demand for countless repetitions of the same instruction

. it may free the teacher of not just routine instructional tasks but also tiresome chores of keeping records of student needs and progress

. an additional varied approach to presentation and drill is made available

. it holds an intrinsic motivational quality that may increase student interest, concentration and retention

Experienced teachers will quickly recognize that these established advantages are thoroughly consistent with sound teaching practices and, skillfully applied, provide considerable support for their efforts to provide students with optimally effective and interesting instruction.

We are confident that the enclosed software will make teaching and learning a little more rewarding and enjoyable for you and your students.

Classroom Management Procedures

Thirty-six students working independently at 16 or more computer stations can easily lead to an almost chaotic confusion during a computer lab period, particularly if finding and distributing each student's working diskette takes too much time at the start. In addition, your ability to retrieve the performance information the program will record for each student depends upon carefully organizing the distribution and collection of the diskettes from the start. For these reasons, and a host of others, it is a good idea to think of classroom management procedures long before the computer lab period actually takes place, and to structure both the computer period and the computer room in ways that will make the period run smoothly and effectively.

In an ideal world each of your students would be able to work independently at a computer workstation, and that situation may exist in some classrooms at some times during the year. But most classes consist of 30 or more students and most computer labs of 16 or fewer workstations; the large number of workstations and the larger number of students require careful record keeping and classroom management. Students must work in a highly structured environment in the lab or they will accomplish much less than you expect them to.

The following outline of suggested procedures will help you create a structured computer lab; after a few periods in the lab you may want to modify the procedures to better suit your own needs.

I. Before the computer lab period.

1. If you are not familiar with the computers in your school, or not entirely comfortable with them, spend some time working with them on your own before you guide your students in using them. Your uncertainty or uneasiness about computers may leave you at a loss at some point in your lesson or be transmitted to those of your students who also feel intimidated by the machine. In either case it will have a negative effect on your lesson.

2. Preview the software you intend to use thoroughly. Learn what it does and how it works. Find out what background information your students will need to run the program --for example, what vocabulary they will have to know. Once 16 groups of students start working and, presumably, asking questions, it will be impossible to decipher anything about the program that you, and they, may find confusing.

3. Number each workstation in the computer lab clearly, so that you and the students can see the numbers easily.

4. The copyright of the enclosed diskettes belongs exclusively to the New York City Board of Education. Any New York City public school employee may make an unlimited number of these diskettes for use in the New York City Public Schools, either in day or evening sessions, without obtaining further permission. Use the DOS diskcopy command to create one set of operating diskettes of the program you plan to use for each student in the lab, plus some extras in case a few of your copies don't function or are lost. Do this for each class with which you intend to use the program. After you have made as many copies as you need, place the original program diskette in a secure place for future duplication.

If it isn't possible to copy one student working diskette for each member of the class, you can use one diskette at each workstation. However, the advantages of individual student diskettes outweigh the disadvantages, and by a wide margin. Diskettes are relatively inexpensive, particularly when they are ordered in large quantities, and you will be able to use them year after year, virtually forever, simply be recopying the original program diskette over them whenever a class completes the program. In addition, providing each student with an individual diskette will prevent certain student generated errors from occurring and make classroom management and student evaluation much easier for you. Refer to the section of this booklet entitled "Software Duplication Guidelines" for copying instructions and for information about obtaining diskettes through a Bureau of Supplies contract.

5. Clearly label each diskette to coincide with one of the workstations--i.e., diskette number 14 must always be used at workstation 14. (Of course, any diskette will work at any station, but for classroom management purposes make sure the students follow this rule strictly.)

6. Divide your class into workstation groups of two, three, or four students, as needed. Look over the student combinations carefully before making final assignments. If you don't feel that a particular combination of students will work well together for an extended period of time, don't allow them to form a group. Once a group has been established, it will be necessary for those students to work together through each of the modules in the program, a procedure that can last for weeks.

7. Assign each group a work station number and keep a close record of the names of the group members and their group number. Each group should work at the same workstation through the entire program, and with the same student operating diskette. Each student operating diskette will eventually hold all of the records of only those students who use that diskette.

   If a student uses a different diskette each time he works, the records will be scattered among them all, and it will be difficult to retrieve them; for 30 or more students it would be too time consuming to be feasible.

8. As an extra precaution, label each diskette with the names of the students who will use it.

II. During the computer lab period.

1. At the beginning of the period have the students sit at their assigned workstations, with no exceptions. Then distribute the diskettes according to their corresponding numbers; the student names on the diskettes should also correspond to the students who are at each workstation.

2. At the end of the period collect the diskettes and store them in numerical order to make their distribution at the next period easier.

3. Once you have established workstation groups, assignments, and procedures, resist any temptation to make changes unless the changes will clearly be to your advantage. As in any classroom, the closer you follow established procedures, the more control you will maintain over the period and the more effective the use of the software will be.

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TOPIC 1: Logic and Statement

Aim: What is logic? What is a statement?

Performance Objectives: The students will be able to:

- describe what is meant by logic.
- define each of the following terms:
  a) sentence
  b) open sentence
  c) variable
  d) statement
  e) domain
  f) solution set
- distinguish between statements and non-statements.
- distinguish between an open sentence and a statement.
- determine the truth or falsity of many statements.
- find the solution set for open sentences given the domain of their variables

Vocabulary: logic, sentence, statement, open sentence, variable, domain, solution set

Materials:

Chalkboard, chalk
Hardware: Apple IIe with a large monitor so that entire class can view the software as the teacher uses it in guiding the lesson.
Software: Elem. Logic-Level 1, Courses by Computers Inc.

Preparation:

Teacher should review and be totally familiar with this software before using it in the classroom.

Motivation:

The program contains built-in motivation since it begins by asking the question "Why study logic?" and then gives convincing reasons for its study. It uses the Little Microman to point out topics, lessons, definitions, concepts, comparisons, i.e.,

1. Why study the topic?

2. Statements, open sentences and negations.

3. Quantified statements

4. The software presents the following two statements that turn out to have the same meaning:

   a. It is false that all bankers are rich.
   b. Some bankers are not wealthy.

5. The use of words is an important part of logic.

Development:

1. To cover statements and open sentences - go to option 1.

2. Three example are given - must show that they have a truth value: namely, true or false; otherwise the statement is an open sentence that can use a variable for the unknown.

3. It is stressed that we be careful of the use of the words "all", "some" or "none."

   a. Example: "She is a movie star" is neither true nor false until "she" is defined. Introduce: variable.

   b. Now present:
      a) x is greater than 5.
      b) He has brown hair.
      c) It is made of metal

      Ask pivotal question: What is the difference between these sentences and those above?
      Elicit: The truth of these sentences cannot be determined without additional information.

      Pivotal question: What are possible answers?
      Discuss the fact that these are determined by the choices which are available. Hence the concept of domain is introduced.

4. The development of the concepts on the software is clear and frees the teacher to give more individual attention.

Application:

Hit spacebar to move to correct answer. Also use the additional applications below.

A) DIRECTIONS

1. State whether each sentence is an open sentence or a statement.

2. For each statement, determine whether it is true or false.

3. For each open sentence, state the variable and replace it to make a true statement.

a) 7 + 3 = 5 + 5
b) The day after Thursday is Wednesday.
c) y + 6 = 18
d) It has three sides.
e) He is six feet tall.
f) The Bronx is in New York.
g) It is a city in Connecticut.
h) 3 times 5 is less than 12.
i) x is an odd number.

B) 1. Given the domain for each variable is the set of whole numbers $\{0,1,2,3,\ldots\}$, find the solution set for each of the following.

a) $x + 7 = 12$
b) $y - 2 = 6$
c) Two times a number is greater than 5.
d) x is less than 4.
e) z is an odd number.

2. For each of the above, write the solution set if the domain is $\{2,4,6,8,10\}$. (The teacher should note the possibility of the empty set as a solution set.)

C) 1. If the domain for each variable is {Monday, Tuesday, Wednesday, March, April, June, September}, find the solution set for each of the following sentences.
a) It begins with the letter M.
b) It is a day that comes after Monday.
c) It contains 30 days.
d) The word contains 6 or fewer letters.
e) It is a day of the week.

Summary:

1. What is logic?

2. Why can't we determine the truth of all sentences?

3. How does the domain of the variable affect the solution set of an open sentence?

4. Give an example of a statement which is true.

5. Give an example of a statement which is false.

6. Give an example of an open sentence which is always true (true for every possible replacement of the variable).

7. Give an example of an open sentence which is never true.

Homework:

1. State whether each of the following is an open sentence, a statement, or neither.

a) The formula for the area of a circle is $A = \pi r^2$.
b) $\pi = \frac{22}{7}$
c) The Greatest Show on Earth.
d) Who is the president of the United States?
e) Zero is neither even nor odd.
f) $x + x = 2x$ whenever x is a number.
g) $x + y = 6$
h) $x + y = y + x$

TOPIC 2: Negation and Truth Tables

Aim: What is the negation of a statement?
How can negation be symbolized using ideas from the study of electricity?
How do we construct a truth table?

Performance Objectives: Students will be able to:

- use a symbol to represent a statement.
- form the negation of a statement in both words and symbols.
- compare the effect of multiple negations using circuits as opposed to verbal statements.
- construct the truth table for negation.
- given an open sentence in symbolic notation using p, determine when the statement that results from substituting given statements for the statement variable is true, false or open.
- given an open sentence and the domain of its variable, negate it and find its solution set.

Vocabulary: negation, truth table

Materials:

One or more copies of "Rocky's Boots" software published by The Learning Company. It is available for the Apple II series, for MS DOS computers and for the Commodore 64.

Motivation:

Load the program "Rocky's Boots" into the computer with a large screen TV or multiple small monitors (to facilitate the demonstration). Choose "Rocky's Challenge" from the main menu and move the cursor to the spare parts rooms. Drag the following items into a fairly empty room: one clacker, 4 NOT gates, and one splitter. You will use these electrically oriented symbols to create various one-way circuits. Touch the cursor to the clacker and show the class what happens. Electricity is flowing into the circuit (shown by the white wire turning orange) and the clacker makes a clacking sound and moves up and down (indicating that electricity has reached it causing it to do its job). Define a clacking clacker on the board as a true result and a motionless clacker as a false result. Define the action of applying electricity to the circuit by moving the cursor over the sensor of the device as a true action and not applying electricity as a false action. Now get one of the NOT gates with the electrified cursor. The sensor turns orange. QUESTION-- "Why has the NOT gate's sensor turned orange? ELICIT-- "Because electricity is flowing into the NOT gate." Now connect the NOT gate to the clacker's sensor. QUESTION-- "If I am applying electricity at one end of the circuit, why is the clacker not clacking?" ELICIT-- Electricity is being prevented from getting to the clacker via the NOT gate." Now take the cursor away from the NOT gate and the clacker starts clacking. QUESTION-- "I'm no longer applying electricity to the NOT gate and yet the clacker is clacking. What's going on?" ELICIT-- "The NOT gate causes the clacker to do the opposite of what it normally does. If I start with electricity flowing <TRUE>, I get no clacking <FALSE>. If I start with no electricity flowing

<FALSE>, the clacker starts clacking <TRUE>. QUESTION-- "Summarize the action of the NOT gate." ELICIT-- "A NOT gate forces you to end up with the opposite truth value from what you started with."

Development

Is the statement "Four times two = 7" true? How could we make this a true statement? (Students may suggest 3 1/2 times two = 7, four plus three = 7, three plus four = 7, etc.) Then ask: Could we make this a true statement without changing any of the words, symbols or numerals of the original statement?

1. Elicit that the insertion of a negative such as not would make this a true statement. For example, the sentence "4 times 2 is not = 7" is true.

   Ask pivotal question: What is the relationship between the original sentence and the new sentence?

   Elicit: One is the negative of the other.
   These statements are called negations of each other.

2. Present these sentences and have the students form the negation of each. Have them determine the truth or falsity of each sentence and of its negation.
   a. The day after Monday is Tuesday.
   b. A square has three sides.
   c. 4 + 7 does not equal 11.
   d. Elephants like to eat peanuts.
   e. Michael Jackson did not sing "Beat it".

   Elicit the fact that, as in 2c and 2e, the negation of a sentence that contains a negative word is the same sentence with the negative word removed.

3. Ask pivotal question: How do we translate a verbal sentence into a mathematical sentence?
   Elicit the procedure of replacing words by symbols. Introduce the symbol ~ for negation. Elicit that symbols may be letters. For example, 3n stands for three times a number.
   In logic, we often use letters to stand for statements. A statement such as "It is raining." may be assigned the letter p. (Note: If more than one statement is used; the letters q, r, s and t are the subsequent letters that can be used.)
   Ask if p is "It is raining.", what is the symbol for "It is not raining."? Elicit ~p. Ask, "What is the negation of ~p?" (~(~p) =p)

   Go back to the "Rocky's Boots" screen and attach a second NOT gate to the clacker and first NOT gate. All of a sudden, the circuit functions as usual, with the clacker clacking when you touch the cursor to the sensor and the rightmost NOT gate. Add a third NOT gate and show the class what happens (works like only one NOT gate was there) and then do it with 4 NOT gates (works like only two were there or like none were there).
   QUESTION-- "Can we express a rule for multiple NOT gates which would be similar for negation symbols?" ELICIT-- "Two NOT gates cancel each other out so that if you have an even number of NOT gates (negations), it is the

same as if you had none. If you have an odd number, you always have an extra, thus, it is like having only one.

4. Introduce the idea of a truth table. Inform the students that in logic we use truth tables to help determine and indicate the truth or falsity of statement. The truth table will have columns and rows. The truth table for negation will begin as follows.

p	~p

Elicit that there are only two possibilities for the statement p. That is, p may be true (T) or p may be false (F). For statements, "maybe" is not allowed. Hence the truth table will continue as follows:

p	~p
T	
F	

Ask pivotal question: What generalization can be made about a statement and its negation?
Elicit: Both cannot be true at the same time. Then complete the truth table.

p	~p
T	F
F	T

5. Present the statement, (1) "She sits in the first seat."
   a. Elicit that it is an open sentence.
   b. Elicit the negation: "She does not sit in the first seat."
   c. Using the class as the domain, elicit the solution set.
   d. Repeat with each of the following.
      (2) "He has blonde hair."
      (3) "She is not wearing a skirt.
      (4) "He is wearing a hat."

6. Translate each of the above (#5) into symbols and then form the negation of each. Elicit:
   (1) p (~p)
   (2) p (~p)
   (3) ~p (p)
   (4) p (~p)

Elicit the concept that the negation of a negation of a sentence is the original sentence.

Applications:

1. Give the negation of each of the following statements and determine its truth or falsity.
   a. $5 = 3 + 2$
   b. April comes after March.
   c. $4 \times 2 > 9$
   d. The Jets are a great football team.
   e. Tom Selleck is not handsome.

2. If p represents "It is raining.",
   q represents "$8 \times 3 = 25$" and
   r represents "I love Mr. T.",

   Write the negation of each of the following in symbolic form.
   a. It is raining.
   b. $8 \times 3 \neq 25$
   c. I do not love Mr. T.
   d. It is not raining.
   e. I love Mr. T.
   f. $8 \times 3 = 25$

3. Complete each of the following truth tables.

a)

p	~p
T	
F	

b)

q	~q
T	
F	

c)

r	~r

d)

p	~p	~(~p)

e)

s	~s	~(~s)	~(~(~s)

4. For each of the following: (1) identify the variable and replace the variable in the original sentence to create a true statement; (2) write the negation of the sentence.
   a. $x + 4 = 6$
   b. $3x = 12$
   c. He is not a basketball star.
   d. She has curly hair.
   e. It is not a planet.
   f. It is a month with 31 days.

Summary:

1. What is the symbol for negation?

2. What is the relationship between a statement and its negation?

3. What is the purpose of a truth table?

4. Give an example of a statement whose negation is true.

Homework:

1. If the domain of each variable is the set of whole numbers less than 10, find the solution set for each of the following.
   a. e is an even number.
   b. e is not an even number.
   c. $x^2 < 9$
   d. $x^2 > 9$
   e. $x^2 = 9$
   f. p is a prime number.
   g. p is not a prime number.
   h. c is a composite number.
   i. $x + x = 2x$
   j. $\frac{n}{n} \neq 1$
   k. $2x + 3 = 12$

2. Write the negation of each of the following.
   a. 1 is a prime number.
   b. $\frac{0}{0} = 1$
   c. 0 is an even integer.
   d. $x > 1$
   e. $y > 1$
   f. 3 is a prime factor of 10.
   g. $\pi = \frac{22}{7}$
   h. $\pi = 3.1416$
   i. $\pi \neq 3.14$ to the nearest hundredth.
   j. The formula for the area of the circle is: $A = \pi D$.
   k. All three angles of a triangle measure 60 degrees.

3. Complete the truth table:

v	~v	~(~v)

4. Let the variables represent the statements as indicated.
   a. r = "The sun is shining."
   b. s = "The color of milk is green."
   c. t = " $\pi = 3\frac{1}{7}$ "
   d. u = "$3 + 4 < 5 - 2$"

   State the negations:
   ~r, ~s, ~t, ~u

TOPIC 3: Conjunctions

Aim: What is a conjunction?

Performance Objectives: Students will be able to:

- describe a conjunction.
- translate sentences using the word "and" into the notation of logic using the symbol .
- determine, given an open sentence in symbolic notation using ^ , whether the sentence that results from substituting given sentences for the $\wedge$ variables is true, false or open.
- compare the effect of conjunctions using circuits as opposed to verbal statements.
- construct the truth table for conjunction.

Vocabulary: Conjunction, compound sentence.

Materials:

One or more copies of "Rocky's Boots" software published by The Learning Company. It is available for the Apple II series, MS DOS computers and for the Commodore 64.

Motivation:

Determine whether each statement is true or false:

a. The day after Wednesday is Thursday and San Francisco is a city in California.
b. 6 is an even number and 8 is an odd number.

Development: (Include pivotal questions):

1. Pivotal questions :
   a. What do these sentences have in common?
   b. How are these sentences different from those we examined when discussing negations?

   Elicit that these are compound sentences.
   Ask: What is the key word in these sentences which makes them compound sentences? (and)
   Elicit the definition: A conjunction is a joining of two statements into one sentence with the word "and."

2. Discuss the motivation exercises and elicit that the first sentence is true but the second is false.
   Elicit that each sentence contains two statements. In the first sentence each of the statements is true; therefore, the sentence is true. In the second sentence, the first statement is true but the second is false; hence, the sentence is false.

3. Ask: What do we need to express a conjunction in symbolic form?
   Elicit that there is a need for two (2) letters and a symbol.
   Elicit: We used "p" in a negation. "p" and "q" will be used in a conjunction. Indicate that the symbol for "and" is "$\wedge$" and it is used as follows: $p \wedge q$.

Let p represent "3 + 4 = 7". Let q represent "Monday begins with an M." Let r represent "Snow is white." Have the students represent the following sentences symbolically.

a. 3 + 4 = 7 and Monday begins with an M.
b. 3 + 4 = 7 and snow is white.
c. Snow is white and Monday begins with an M.
d. 3 + 4 = 7 and snow is not white.
e. Monday doesn't begin with an M and 3 + 4 ≠ 7.

Load the program Rocky's Boots into a computer with a large screen TV or multiple small monitors (to facilitate the demonstration). Choose Rocky's Challenge from the main menu and move the cursor to the spare parts rooms. Drag the following items into a fairly empty room: 1 clacker, 1 AND gate, 2 NOT gates, and 1 splitter. You will use these electrically oriented symbols to create various one-way circuits. Since you've already shown them how the clacker, cursor, splitter, and NOT gate work from the previous lesson, all that remains is to demonstrate the AND gate. Connect an AND gate to a clacker and try the following experiments. First touch the electrified cursor to the bottom sensor of the AND gate. Carefully move the cursor around to touch the top sensor of the AND gate. In each case, the individual sensor will turn orange indicating electricity is entering it, but the clacker will not sound. Touch the cursor to both sensors simultaneously and observe the clacker clacking. Take the sensor away again and repeat. QUESTION "How does this AND gate work?" ELICIT-- "Electricity passes through the AND gate to the clacker only when both sensors are electrified (made TRUE)." QUESTION-- "In terms of truth values, how does the AND gate work in this circuit?" ELICIT-- "The only way a clacker clacks (TRUE result) when attached to an AND gate is when both inputs of the AND gate are electrified (both parts must be TRUE).

4. Elicit that a truth table for a conjunction will have three (3) columns.

In constructing this truth table, we must consider all possibilities. Elicit that as each statement can be true or false, there are four possibilities to consider. Hence the truth table starts as follows.

p	q	$p \wedge q$

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

From this we elicit the complete table:

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

5. If p represents "Spring has 4 letters.", q represents "8 - 4 = 2", and r represents "He is 6 feet tall.", elicit the sentences that correspond to each of the following and discuss whether each is true, false or an open sentence.

   a. $p \wedge q$
   b. $q \wedge r$
   c. $r \wedge p$
   d. $\sim p \wedge q$
   e. $r \wedge \sim p$

   Have students note that only those conjunction sentences in which both parts are true are true sentences.

6. Ask, as a medial summary: When is a conjunction a true statement?

Applications:

Use the following sentences for questions 1 and 2.
Let p, q, r and s be as follows.
p: Roses are red.
q: 16 > 15
r: 25 is an even number.
s: Dave Winfield plays first first base.

1. Using the above, translate each into a sentence and determine whether it is true or false.

   a. $p \wedge q$
   b. $q \wedge r$
   c. $p \wedge s$
   d. $q \wedge \sim r$
   e. $q \wedge \sim s$
   f. $\sim p \wedge q$

2. Using the above, translate each sentence into symbolic form.

   a. Roses are red and Dave Winfield plays first base.
   b. 16 > 15 and roses are red.
   c. 16 > 15 and 25 is an even number.
   d. Roses are not red and 16 > 15.
   e. 25 is not an even number and Dave Winfield doesn't play first base.
   f. 25 is an even number and 16 is less than 15.

3. Complete the following truth tables.

a)

p	q	$p \wedge q$

b)

r	s	$r \wedge s$

c)

p	q	$\sim p$	$q \wedge \sim p$

d) Construct a truth table for $(\sim(q)\ \wedge\ (\sim p)$

4. Allow groups of students to attach a NOT gate to one sensor of the AND gate and observe the results. Add a pair of NOT gates and observe the results.

Summary:

1. What is a conjunction?

2. When will a conjunction be a true statement?

3. How many possibilities must be considered when constructing a truth table for a conjunction sentence of two clauses?

4. Give an example of a conjunction which is always false. $(p \wedge \sim p)$

5. What is the relationship between what you observed in application #3, the truth tables, part c and d, and application #4? Explain.

Homework:

1. If the domain of each variable is the set of rational numbers (numbers expressable as the quotient of two integers) find the solution set for each of the following.,

a. $2x + 3 = 12$
b. $2x - 3 = 12$
c. $2x + 3 = -12$
d. $2x - 3 = -12$
e. $2x + 3x = 12$
f. $2x - 3x = 12$
g. $n^2 = 9$
h. $|y| < 4$ Note: $|n| = \left\{\begin{array}{l} n \text{ if } n \geq 0 \\ -n \text{ if } n < 0 \end{array}\right\}$, $|3| = |-3| = 3$

i. $z^2 = 2$

2. Let the variables represent the statements as shown.
p = "The Yankees were a great team."
q = "Swimming is an excellent sport."
r = "A woman was never president of the U.S."
$s = (2.5)^2 = 6.5$

Express in words and state the truth value.

a.	$p \wedge q$	e.	$\sim p \wedge (\sim s)$
b.	$p \wedge (\sim q)$	f.	$s \wedge (\sim r)$
c.	$r \wedge s$	g.	$p \wedge q$
d.	$\sim q \wedge (\sim r)$	h.	$\sim p \wedge s$

3. Fill in the truth table.

a)

q	r	$q \wedge r$	$\sim(q \wedge r)$
T			
F			
T			
F			

b)

q	r	$\sim q$	$\sim p$	$\sim q \wedge (\sim s)$
T				
F				
T				
F				

4. Do the following have the same truth value? If not, show by example.

a. p , $\sim(\sim p)$
b. $\sim p \wedge q$. $p \wedge (\sim q)$
c. $\sim(p \wedge q)$, $\sim p \wedge (\sim q)$
d. $p \wedge q$, $q \wedge p$

5. Is conjunction commutative?

6. Does negation distribute over conjunction? (In other words, does $\sim(p \wedge q)$ have the same truth value as $\sim p \wedge \sim q$ for all possible truth values of p and q?)

TOPIC 4: Disjunctions

Aim: What is a disjunction?

Performance Objectives: Students will be able to:

- describe a disjunction.
- translate sentences using the word "or" into the notation of logic using the symbol V.
- determine, given an open sentence in symbolic notation using V, whether the sentence that results from substituting given sentences for the statement variable is true, false or open.
- compare the effect of disjunctions using circuits as opposed to verbal statements.
- construct the truth table for disjunction.

Vocabulary: disjunction, compound statement, truth value

Motivation:

There are two clubs in the school with the following requirements for memberships:

HONOR SOCIETY	SERVICE CLUB
85% average	service to the school
and	or
service to the school	service to the community

George has a 78% average, and he has been a monitor in the school library for two years. Is he eligible to join these clubs?

Materials:

One or more copies of "Rocky's Boots" software published by The Learning Company. It is available for the Apple II series, for MS DOS computers, and for the Commodore 64.

Development:

1. Elicit that George is eligible to join the Service Club but not the Honor Society. The requirements for the Honor Society involve a conjunction; the requirements for the Service Club involve another connective, namely "or".
   Elicit from the students some simple sentences containing "or". Name these sentences "disjunctions" and elicit the following definition:
   A disjunction is a compound sentence in which two statements are joined by the word "or".

   Discuss the truth or falsity of each part of the sentences and then the entire sentences.

   Consider:
   a. April has snow or "May" contains three letters.
   b. 5 + 7 = 12 or 5 + 7 = 13.
   c. This teacher is great or Michael Jackson is a singer.

Elicit that if either part is true, then the entire sentence is true.

Load the program Rocky's Boots into a computer with a large screen TV or multiple small monitors (to facilitate the demonstration). Choose Rocky's Challenge from the main menu and move the cursor to the spare parts rooms. Drag the following items into a fairly empty room: 1 clacker, 1 AND gate, 1 gate, 2 NOT gates, and 1 splitter. You will use these various electrically oriented symbols to create various one-way circuits. Only the OR gate has not been demonstrated in a previous lesson. Attach an OR gate to a clacker and perform the following experiment. Touch one sensor of the OR gate with the electrified cursor and then the other. Then touch both simultaneously. In each case, the following sensor(s) will turn orange indicating flow of electricity and the clacker will sound. QUESTION-- "How does this OR gate work?" ELICIT-- "Electricity passes through the OR gate to the clacker as long as at least one sensor is electrified (made TRUE)." QUESTION-- "In terms of truth values how does the OR gate work in this circuit?" ELICIT-- "The only way a clacker clacks (true result) when attached to an OR gate is when at least one input of the OR gate is electrified (at least one part must be true).

2. Pivotal Question: If the symbol for disjunction is V, how would a disjunction be represented symbolically? Elicit pVq.
   Using p, q, r and s, have the students represent each of the following symbolically and determine the truth or falsity of the compound sentence.
   p: "John" is a boy's name.
   q: 6 < 12.
   r: 14 is an odd number.
   s: All apples are red.

   a. "John" is a boy's name or 6 < 12.
   b. 14 is an odd number or all apples are red.
   c. 6 < 12 or all apples are red.
   d. "John" is a boy's name or 14 is not an odd number.

3. Reverse the process. Present the following symbolic sentences and have the students translate them into words. (Use the same statements.)
   a. pVq
   b. pVr
   c. rVq
   d. rVs
   e. pV~(q)
   f. ~rV~(s)
   Ask, as a medial summary: When is a disjunction a true statement?

4. Develop a truth table. Have students recall the truth table for conjunction. Elicit all the entries.

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

From this, elicit that for disjunction we must consider the same possibilities for p and q.
Ask pivotal question: What, if anything, will be different in the disjunction truth table? The students should note that only the last column is different and should read pVq. Elicit the table.

p	q	pVq
T	T	T
T	F	T
F	T	T
F	F	F

Have the students note the difference in the entries in the last column.
Ask: When is a disjunction true? (When at least one statement is true.)
When is a disjunction false? (When both statements are false.)
Combine the truth table for rVs and r∧s in one as follows.
Ask: How many columns are needed? (4)
Elicit the headings and write the truth table on the board. Elicit all the entries as indicated.

r	s	rVs	r∧s
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

5. Present the sentence: He is seven feet tall or sits in the first row.
Ask: Can this sentence be judged true or false? Why not?
Elicit that this is an open sentence. Replace the "he" with different names and elicit that the sentences now become disjunctions whose truth value can be determined. Present the following sentences and ask the students whether they are true, false or open.

Use p: It is red.
r: x + 5 = 2
q: 6 + 4 = 8
s: She has long hair.

a. pVq
b. pVr
c. rVs
d. qVr
e. pVs

Elicit the variables in each of the above.
Again ask: When is a disjunction a true statement?

Applications:

Use the statements p, q, r and s as follows for the exercises in part 1 and 2.

p: Basketballs are round.
q: Horses have three legs.
r: $18 = 24 - 6$
s: Falling snow is green.

1. Using the statements above, write each of the following in symbolic form and determine the truth value of each sentence.
   a. Basketballs are round or horses have three legs.
   b. Horses have three legs or falling snow is green.
   c. Falling snow is green or $18 = 24 - 6$.
   d. Falling snow is green or basketballs are round.
   e. Basketballs are not round or horses have three legs.
   f. Falling snow is not green or $18 \neq 24 - 6$.
   g. Horses have three legs or $18 \neq 24 - 6$.

2. Using the statements p, q, r and s above, write each of the following in words and determine its truth value.
   a. $p \vee q$
   b. $p \vee r$
   c. $r \vee s$
   d. $q \vee s$
   e. $q \vee (\sim r)$
   f. $r \vee (\sim p)$
   g. $q \vee (\sim s)$
   h. $s \vee (\sim r)$

3. Complete the following truth tables.

a)

p	q	pVq
T		
T		

b)

r	s	r∧s	rVs
T			
T			
	T		
	F		

p	q	~p	qV(~p)

p	q	~p	~q	(~p)V(~q)

4. Allow a group of students to attach a NOT gate to one sensor of the OR gate and observe the results. Add a pair of NOT gates and observe the results.

Summary:

1. What is a disjunction?

2. When is a disjunction a true statement?

3. What is the symbol for disjunction?

4. Give an example of a disjunction which is always true.(pV(~p))

   Is there a relationship between what you observed in application #4, and the truth tables in parts c and d of application #3?

Homework:

1. Let each variable have the set of whole numbers for its domain. Find the solution set for each of the following sentences.

   a. (x ∧ 4 ~ (x<10)
   b. (y is odd ∧ (y is a multiple of 3) ∧ (y<20)
   c. (z is even) ∧ (z is a multiple of 3) ∧ (z<20)
   d. (w<4)V (w<10)
   e. (u<4)V(u+u=10)
   f. ~(n>4)V(n+2=12)

2. Let each variable have the set of integers for its domain. Find the solution set for each of the following statements.

   a) (x<4) ∧ (x^2=25)
   b) (y+1=7)V(2y-3=15)
   c) ~(x<4) ∧ (x^2=25)
   d) (w>-3) ∧ (w<3)
   e) (w>-3) ∧ ~(w<3)
   f) [(w>-5)∧(w<5)] ∧ (w^2>1)

3. Complete the truth table.

   a)

p	q	~p	~q	~p∧(~q)	pVq	~(pVq)
T						
F						
T						
F						

b)

p	q	~p	~q	pVq	p^q	~(p^q)
T						
F						
T						
F						

c) Try to express pVq in terms of p, q and the connectives ^ and ~. (Hint: see 3a)

d) Try to express p^q in terms of p, q and the connectives V and ~. (Hint: see 3b)

4. Prepare a truth table for
   a) p^q
   b) ~(p^q)

5. Prepare a table containing both p^(qVr) and (p^q)V(p^r) as column headings. What does this table say about ^ distributing over V?

6. By means of a truth table decide whether V distributes over ^. (See question 5.)

TOPIC 5: Contrapositives of Conditionals

Aim: What is the contrapositive of a conditional statement?

Performance Objectives: Students will be able to:

- describe the contrapositive of a conditional statement.
- write the contrapositive of a given conditional statement of form p ---->q.
- determine the truth value of a contrapositive, converse and inverse of a given conditional statement whose truth value has been determined.
- select from several conditional sentences the sentence which is logically equivalent to the given sentence.

Vocabulary: Contrapositive, logically equivalent.

Materials:

Chalkboard, chalk
Hardware: large monitor with an Apple IIe computer
Software: Elementary Logic - Level 1, Courses by Computer

Motivation:

Given the conditional: If it is a square, then it has four sides. What is its relationship to the following sentence: If it doesn't have four sides, then it is not a square.

Development:

1. Use of the software - it is user friendly - it asks for the students' name. It has Microman - a little man who indicates important concepts, definition, etc.

2. Go to lesson 5 - converse, inverse and contrapositive. It provides a review of the converse and inverse and then a move into the concept of contrapositive.

3. The program goes through the definitions, and the visual nature of highlighted concepts is excellent.

P	Q	~P	~Q	P⟶Q	Q⟶P	~P⟶~Q	~Q⟶P

This Truth Table can be used as the medial summary for the lesson. The relationship between P--->Q and ~P --->Q on the Truth Table is shown.

Applications:

Give several problems to compare logically equivalent statements.

If today is Tuesday, then tomorrow is Wednesday.

a. If tomorrow is Wednesday, then today is Tuesday.
b If 3 is not odd then 5>3.
c. If 5>3 then 3 is not odd.
d. If 3 is odd then 5 is not greater than 3.

Summary:

1. How is the contrapositive of a statement formed?

2. When are two statements logically equivalent?

Homework:

1. State the contrapositive of each of the following.
   a) If the integer is one more than some even integer, then it is an odd integer.
   b) If two angles are right angles, then they are congruent.

   c) If the product of two integers is an odd integer, then both factors are odd integers.
   d) If $x = 5$, then $x^2 = 25$.
   e) If $y > 7$, then $y > 2$.
   f) If $\triangle$ ABC is a right triangle with c being the hypotenuse, then $a^2 + b^2 = c^2$.
   g) If it is a legal holiday, then school is closed.

2. What is the converse and contrapositive of each of the following?

   a) $p \longrightarrow s$
   b) $\sim p \longrightarrow q$
   c) $r \longrightarrow \sim s$
   d) $\sim u \longrightarrow \sim v$
   e) If it is hot, then I perspire.
   f) If I am hungry, then I eat.
   g) If the triangle is equilateral, then it is isosceles.

3. Which of the following is logically equivalent to:
   "If a person drives while intoxicated, he will be sent to jail."

   a) If a person does not drive while intoxicated, he will not be sent to jail.
   b) If a person is not sent to jail, then he did not drive while intoxicated.
   c) A person will be sent to jail if he drives while intoxicated.
   d) If a person is sent to jail, then he drove while intoxicated.
   e) If a person drives while intoxicated, he will not be sent to jail.

4. Using the connectives V,Δ,~,——> as binary operations on statements, decide whether these have the same truth values, and state the kind of distributive properties if any.

a) p ∧ (qVs), (p ∧ q) V ((p ∧ s)
b) p -> (qVs), (p->q) V (p->s)
c) (pVq) ->s, (pVs) -> (pVs)
d) ~(p ∧ s), (~pV~s)

TOPIC 6: Evaluating Algebraic Expressions

Aim: How do we evaluate algebraic expressions?

Performance Objectives: Students will be able to:

- state what is meant by an algebraic expression.
- state what is meant by evaluating an algebraic expression.
- evaluate algebraic expressions using powers and parentheses.

Vocabulary: algebraic expression, exponent, factor

Materials:

Chalkboard, chalk
Hardware: A large monitor hooked up to an Apple IIe computer
Software: Plato Educ. software - Elementary Algebra-Polynomials-Disk I

Motivation:

1. According to the radio, the temperature is 20 degrees Celsius. What is the temperature in Fahrenheit degrees? (The Fahrenheit temperature is 32 degrees more than nine-fifths of the Celsius reading.)

2. Write an expression involving the variable c which could be used to find Fahrenheit equivalents for any Celsius temperature.

Development:

1. Apply the software - an interesting device using the MicroMan to show how to evaluate an algebraic expression. Go to Addition Option.

2. The letters in 2X - 3 and $\frac{Y}{4} - 2$ are called variables.

   a. We can evaluate algebraic expressions by replacing the variable with the value

      - $2X - 3$
      - $2(5) - 3$
      - 7

2. The presentation by Microman is very clear and would allow the teacher to circulate and give individual help.

a. $Y/4 - 2$

$\Downarrow$

b. $8/4 - 2$

$\Downarrow$

c. $2 - 2 = 0$

3. The program is user friendly - it allows the user or class to put a name in for the user, and Microman addresses the user.

   a. Exponents are used - review this concept.

      - Example: Replace D with -2
      - $D^2 - 3D$ $(-2)^2 - 3(-2) = 4 + 6 = 10$
      - Show on chalkboard if necessary.

4. Use of parentheses.
   $3(X + 2)$ if $X = 5$

5. Introduce exponents as a short way of indicating repeated multiplication with the same factor. Show 4^3 means $4 \times 4 \times 4$. Define: X^n is the product of n factors, each equal to X.

6. Demonstrate:

$$\text{a)}\quad 2 \times 4^3 + 7 = 2 \times 4 \times 4 \times 4 + 7$$
$$= 2 \times 64 + 7$$
$$= 128 + 7$$
$$= 135$$

$$\text{b)}\quad (2 \times 4)^3 + 7 = 8^3 + 7$$
$$= 8 \times 8 \times 8 + 7$$
$$= 512 + 7$$
$$= 519$$

7. Expand order of operations to include exponents, as follows:
   a) Parentheses
   b) Exponents
   c) Multiplication and division, in order, from left to right.
   d) Addition and subtraction, in order, from left to right.

Applications:

1. Evaluate each of the following if $a = 8$, $b = 4$, $d = 3$, $x = 6$, $y = 1$, $z = 5$.

   a) $3x + 7$ b) $2dx$
   c) $4y - 2$ d) $7y + 4b - d$
   e) $3(2b - z)$ f) $\frac{3x - 2}{b}$

2. Evaluate each the following if a, b, d, x, y and z have the same values as in application 1.

a) d^4 b) $3b^2$

c) $(3b)^2$ d) $2a^2 + a$

e) $4xb^3$ f) $\frac{d}{2}(a + b)$

3. a) How can we write 3xxyyz using exponents?
b) Evaluate the above expression if $x = 2$, $y = 4$, $z = 5$.

Summary:

1. Explain the use of parentheses in algebraic expressions.

2. State the complete convention for the order of operations.

3. What is an algebraic expression?

Homework:

1. Find the numerical value of the following algebraic expressions if $a = 4$, $b = 8$, $x = 3$, $y = 7$ and $z = 5$.

a) $\frac{11+a}{xz}$ b) $4a - 3x$

c) $2b + y - (z - 1)$ d) $4(b - x) + 2y$

e) $3 + \frac{b}{a} + \frac{3z}{x}$ f) $\frac{2(x+z)+a}{a+b}$

g) b^2 h) $5a^3xy$

i) x^4 j) $3a^2 + b^2$

k) $3x^2$ l) a^3xy^2

m) $3z^2xb$ n) $x^2 + 5x + 4$

2. Express in words:

a) $\frac{a+b+c}{3}$ b) $a^2 + b^2 = c^2$ c) $\frac{2a+b}{3}$

3. Evaluate and tell which is greater:

a) 2^5 or 5^2 ?

b) 7^8 or 8^7 ?

4. When is the logical sentence: $(r \vee q) \wedge (r \vee s)$

a) true b) false

TOPIC 7: Addition and Subtraction of Monomials

Aim: How do we add and subtract monomials?

Performance Objective: Students will be able to:

- identify: a monomial, coefficients, variables, exponents
- state the procedure for addition and subtraction of monomials.
- add and subtract like monomials.

Vocabulary: monomial, literal factor, like monomials, unlike monomials, coefficients, base, exponent, polynomial, terms of an algebraic expression.

Materials:

Chalkboard, Chalk
hardware: large monitor connected to an Apple IIe computer
software: Plato - Polynomials Disk 1

Motivation:

Sharon paid 3x dollars for a computer and 2x dollars for software and accessories. Write the simplest possible expression for the total cost.

Development:

1. Use of the software - define monomial on the chalkboard - use of the distributive property. Pick "Addition" and when program begins, use control-J to get "Addition of Monomials."

2. On the same piece of software choose 2B and control-J to get subtraction.

3. Have students work at the chalkboard. There is immediate reinforcement using the computer. Have students take turns at chalkboard and computer.

4. At the chalkboard make sure exponents are reviewed.

   a. $7Y^2 - 2Y^2 = (7-2)Y^2 = 5Y^2$

5. Develop the rules for addition and subtraction of like monomials: To add like monomials, add their numerical coefficients and multiply this sum by their common literal factor. To subtract like monomials, change the sign of the subtrahend to its opposite and add.

Applications:

1. Name the numerical coefficient of each of the following terms:

   a) $8x$
   b) $5y$
   c) $\frac{1}{2}x$
   d) $-3z$
   e) b

2. Name the base of each of the following terms :

a) $3x^5$ b) x^4

c) $2y^3$ d) 4^2

3. Name a) the numerical coefficient
b) the literal factor
c) the variables of the following terms:

a) $5s$ b) $7y^2$

c) $3xy^2$ d) $3x^2$

e) $-8x^2y^2$ f) $9y^2$

g) $-8xy^2$ h) $3x^2y$

4. Which of the terms in application 3 are like terms?

5. Combine like terms:

a) $16x^2 - 2x^2$ b) $2a + 7a$

c) $-5xy + 3xy$ d) $\begin{array}{r} +9x \\ \underline{-9x} \end{array}$

6. Compute the difference:

a) $\begin{array}{r} +8y \\ -\ \underline{+3y} \end{array}$ b) $\begin{array}{r} -5x \\ -\ \underline{-3x} \end{array}$

c) $\begin{array}{r} 14t \\ -\ \underline{-t} \end{array}$ d) $\begin{array}{r} +4xy \\ -\ \underline{+4xy} \end{array}$

e) $\begin{array}{r} b \\ -\ \underline{-3b} \end{array}$ f) $\begin{array}{r} 2x \\ -\ \underline{+5x} \end{array}$

Summary:

1. What is a term?

2. What are like terms?

3. What are opposites?

4. State the rule for combining like terms.

Homework:

1. Which of the following are monomials?

a) $4x - 2w$ b) $3x^2$ c) $-5xy$

d) $31x^2y^2z^2$ e) $3x^2 - 2y$

2. **Name the numerical coefficients of the following monomials:**

a) $4x$ b) $-10z$
c) b d) $7x^2y$
e) abc f) $-x$

3. **Name the variables:**

a) $4x$ b) $-10z$
c) b d) $7x^2y$
e) abc f) $-x$

4. **What is the exponent in each of the following monomials?**

a) x^5 b) $-21y^2$ c) $3x^4$
d) $3x$ e) $-4x^3$

5. **Add:**

a) $\begin{array}{r} +12x^2 \\ \underline{+\ 5x^2} \end{array}$ b) $\begin{array}{r} +32y \\ \underline{-18y} \end{array}$ c) $\begin{array}{r} +22bc \\ \underline{-52bc} \end{array}$

d) $\begin{array}{r} -43xy^2 \\ \underline{-13xy^2} \end{array}$ e) $\begin{array}{r} -8r \\ \underline{+9r} \end{array}$ f) $\begin{array}{r} -12mn \\ \underline{+12mn} \end{array}$

6. **Subtract:**

a) $\begin{array}{r} +4y \\ \underline{-\ +y} \end{array}$ b) $\begin{array}{r} -5st \\ \underline{-\ -2st} \end{array}$

c) $\begin{array}{r} 0 \\ \underline{-\ +3x} \end{array}$ d) $\begin{array}{r} 2x^2 \\ \underline{-\ x^2} \end{array}$

e) $\begin{array}{r} +xy \\ \underline{-\ +3xy} \end{array}$ f) $\begin{array}{r} -4x^2y^3 \\ \underline{-\ -4x^2y^3} \end{array}$

7. **Solve:**

a) $3(2x + 3) - 6x = x$
b) $4(5 - y) + 4y = y$

8. **The dimensions of a rectangular garden are 3x and 8x. If you surround the garden with a fence, how long will the fence be?**

TOPIC 8: Multiplication of Monomials

Aim: How do we multiply monomials?

Performance Objective: Students will be able to:

- state the rules for multiplying monomials with regard to coefficients, variables and exponents.
- multiply monomials containing varied types of coefficients and positive integral exponents.

Vocabulary: exponent, coefficient, base, factor, monomial.

Materials:

Chalkboard, chalk
Hardware: A large monitor hooked up to an Apple IIe computer.
Software: Integers (a contest) by SVE Microcomputer Software.

Motivation:

1. Write an equivalent expression without exponents.
   a) $12x^5$ b) $10x^2y$
2. Which is larger: $(2^3)(2^4)$ or 2^{12}?

Development:

1. Use motivation. Why are exponents used? Review that the use of exponents is a short way to show multiplication using the same factor.

2. Review that in examples such as x times y = xy, the fact that there is no operational symbol between the x and y indicates x is multiplied by y.

3. How can we use the commutative and associative properties to simplify (5c) (4c)? Develop that
   $(5c)(4c) = (5)(4)(c)(c) = 20c^2$

4. Similarly, develop that
   $(2x)(4x)(6xy) = (2)(4)(6)(x)(x)(x)(y) = 48x^3y$

5. Write $(x^3)(x^2)$ without exponents. Show that
   $(x^3)(x^2) = (x)(x)(x) \cdot (x)(x) = x^5$. Generalize:
   $(x^a)(x^b) = x^{a+b}$.

6. Do application 1.

7. Have the students note that in application 1a and 1b the base is the same as in 5 above. How do we multiply powers of the same base?

Elicit that to multiply two powers of the same base, the product has the same base and the exponent is the sum of the factors. Emphasize that the rule applies only when the base is the same:

$x^2 y^3$ cannot be simplified because the bases of x^2 and y^3 are different.

8. Use software to practice multiplication of signed numbers. It is a speed test since most students know the rules, but first present to students:

   What is the product of $(-6x^2 y)$ and $(+3xy)$? Develop that

   $(-6)(x)(x)(y)(+3)(x)(y) = (-6)(+3)(x^2)(x)(y)(y) = -18x^3 y^2$

   Elicit that to multiply monomials,

   a) Multiply numerical coefficients using the rules for multiplication of signed numbers;
   b) Add exponents where the base is the same.

9. Do application 2.

Applications:

1. Compute the product of the following monomials:

   a) $(2a)(5a)$
   b) $(4x)(5x)(3x)$
   c) $(3xy)(4y)(5xy)$
   d) $b^2 b^4$

2. Compute:

   a) $(6)(-2a)$
   b) $(-4r)(5rs^2)$
   c) $(-2b^3)(-3b^5)$
   d) $(+7ab)(-2c)$
   e) $(+4x^2 y^3)(7x^3 y^4)$
   f) $(+5a^2 b^3)(3b)$
   g) $(-3xy^2)(-4x^2 y^3)$
   h) $(-3)(+4x)(-3x)$
   i) $(-2x)^2$
   J) $(+5x^4)(-2x)(-b^3)$

Summary:

1. How many terms are in the product of monomials? Have students conclude that when any number of monomials are multiplied, the product is a monomial.
2. What is the value of $(2x^5)^3$?

Homework:

1. Find the product:

a) $(+5)(-2x)$

b) $(+4x)(-3y)$

c) $(-3r)(-s)$

d) $y^6 y^4$

e) $a^2 a$

f) $t^5 t^3 t$

g) $(4y)(2y^4)$

h) $(-5x^2)(3xy)$

i) $(-2r^2t^2)(-5r^2t^3)$

J) $(+4z)(-3z^4)(-4z^3)$

2. Find the product:

a) $(4x)(-5y)$

b) $(+6r)(-3st)$

c) $(3ab)(6cd)(-2e)$

d) $(20x^3)(-4x^6)$

e) $(-8a^4b)^3$

f) $(5x^5y^4)(-6x^4y^3)(4x^2)$

g) $(+4x^2)^3$

h) $(-3y)^2$

TOPIC 9: Negative and Zero Exponents

Aim: What is the meaning of a negative exponent?
What is the meaning of a zero exponent?

Performance Objectives: Students will be able to:

- recall the rule for division of monomials.
- explain the meaning of a negative exponent.
- explain the meaning of a zero exponent.
- rewrite monomials containing negative exponents as monomials with positive exponents.
- use a spreadsheet to do calculations and make comparisons.

Vocabulary: exponent, base, power

Materials:

- chalkboard, chalk
- hardware: computer lab or computer with a large monitor
- software: any electronic spreadsheet compatible with your computer

Motivation:

1. Present students with the following numbers:
   a. 4
   b. 16
   c. 9
   d. 125
   e. 1000

2. For each, have the student come up with an appropriate base raised to an appropriate power that gives an equivalent answer.
   Note: students should be reminded that not all the numbers are perfect squares, but they may be perfect cubes, etc.

Development (Include pivotal questions):

1. Provide students with an electronic spreadsheet loaded into one or more computers organized as follows:
   a. Column A has the five numbers listed vertically.
   b. Column B has each example's reciprocal in decimal form.
   c. Column C is blank (reserved for student entry).
   d. (OPTIONAL) Column D checks Column C for correctness and generates a comment using a conditional.

2. Instruct to enter the same base raised to a different power into Column C attempting to match Column B.

3. Students can keep track of these exponents in their notebook and compare their correct responses.

4. Elicit: Raising a positive whole number to a negative power seems to generate a decimal.

5. Question - What is the relationship between the exponents arrived at doing the motivation and the ones we get looking for reciprocals? Elicit - The exponents are additive inverses thus opening up the possibility of getting negative exponents in certain situations.

6. Give the student a rectangle whose area is known to be 2^5. Using the rules for division of monomials involving exponents have them find the width when the length is:

   1. 2^3
   2. 2^5
   3. 2^7

7. After the students arrive at 2^2, 2^0, and 2^{-2}, elicit: 2^{-2} is the reciprocal of 2^2 thus it must equal $\frac{1}{2^2}$.

8. Recall the rule for dividing powers with like bases:

   $$\frac{x^a}{x^b} = X^{a-b} \quad \text{where x is not equal to zero.}$$

   Suppose a=b; then:

   $$\frac{x^a}{x^b} = \frac{x^a}{x^a} = x^{a-a} = x^0$$

   But $\frac{x^a}{x^a}$ is a division of a number by itself.

   Question- What is the result when you divide a number other than zero by itself? (1)

   Therefore $x^0 = 1$ when x is not equal to zero. Since $x^0 = 1$ is based on division, 0^0 is undefined.

9. Using the definition $x^0 = 1$ where x is not equal to zero permits to observe that:

   $7^0 = 1$

   $(-7)^0 = 1$

   $(7x)^0 = 1$

   $(-7x)^0 = 1$

However, $7x^0 = 7$ because $7x^0 = 7(x^0) = 7(1) = 7$.

10. Do application 1.

11. By the division rule, $\frac{x^5}{x^7} = x^{5-7} = x^{-2}$

But $\frac{x^5}{x^7} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x\ x} = \frac{1}{x^2}$

Therefore, $x^{-2} = \frac{1}{x^2}$

We have the following definition: $x^{-a} = \frac{1}{x^a}$ where x is not equal to zero.

12. Using the above definition and remembering the rule for multiplying powers of the same base $x^a . x^b = x^{(a+b)}$, observe that

$$3^2 \times 3^{-5} = 3^2 \times \frac{1}{3} = \frac{3^2}{3} = \frac{3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{3 \times 3 \times 3} = 3^{-3}$$

13. Do applications 2,3, and 4.

Applications:

1. Find the value of each of the following expressions.

a) b^0, $(b = 0)$ b) -5^0

c) 2^0 d) $(.2)^0$

e) $(2^5)0$ f) $2x^0$, $(x = 0)$

g) $(2x)^0$, $(x = 0)$ h) $\left(\frac{1}{8}\right)^0$

i) $\left(\frac{7}{9}\right)^0$

2. Compute each of the following.

a) 3^{-1} b) 4^{-2}

c) 2^{-3} d) 8^{-2}

e) 2^{-1} f) y^{-2} if $y = \frac{3}{4}$

3. Write an equivalent expression using only positive exponents.

   a) a^{-9} b) x^{-3} c) b^{-1} d) y^{-4}

4. Using laws of exponents, evaluate or simplify:

   a) $X^{0} \times X^{5}$ b) $4^{3} \times 4^{-2}$

   c) $2^{-3} \times 2^{-4}$ d) $4^{3} - 4^{0}$

   e) $7^{-3} - 7^{-5}$ f) $(x^{-3})^{2}$

Summary:

1. If $y \neq 0$, find the value of y^{o}.
   What is the value of any number or variable (base $\neq 0$) having an exponent of zero?

2. Express y^{-2} without using a negative exponent.
   How can we change a variable with a negative exponent to an equivalent expression with a positive exponent?

3. What is the product of y^{n} and y^{-n} ?

Homework:

1. Write each of the following numbers without using any exponents.

   a) 2^{o} b) 6^{-2}

   c) 8^{o} d) 5^{3}

   e) 7^{-1} f) -3^{o}

   g) -9^{-1} h) -3000^{-1}

   i) 16^{o} j) -1^{4}

2. Write each of the following expressions using only positive exponents.

   a) x^{-8} b) $\frac{1}{x^{-6}}$ c) $(3xy)^{-2}$

   d) x^{-1} e) $(3x^{-3})^{2}$ f) $\left(\frac{x}{y^{3}}\right)^{-2}$

3. Using Laws of Exponents evaluate:

a) $6^5 - 6^2$ b) $2^4 \times 2^6$ c) $3^{-4} - 3^5$

4. Write an equivalent expression without a denominator. (other than 1)

a) $\frac{1}{a}$ b) $\frac{bh}{2}$

c) $\frac{h(b+b')}{2}$ d) $\frac{z}{2x^2y}$

5. Evaluate each expression when x = 2, y = 3, z = 2.

a) $\frac{6x^2y^5}{2xy^3}$ b) $\frac{4x^2yz^3}{4x^2yz^3}$ c) $\frac{2xy^3}{6x^2y^5}$

TOPIC 10: Subtraction of Polynomials

Aim: How can we subtract polynomials?

Performance Objectives: Students will be able to:

- define and identify a polynomial.
- recognize like terms from two or more polynomials.
- recall the rules for subtraction of monomials.
- apply rules for subtraction of monomials to subtraction of polynomials.

Vocabulary: polynomial, binomial, trinomial, degree of a polynomial, subtrahend, minuend, difference, descending order, ascending order, like terms, additive inverse.

Materials:

software: "Elementary Algebra: Polynomials" published by Plato Educational Courseware

hardware: Apple II, IIe Computers Monitor and Disk Drive

Motivation:

How would you subtract $+3x^2 + 5x - 1$ from $4x^2 + 2x + 3$?

Development:

1. To subtract polynomials, we use a procedure similar to that in subtracting monomials. We add the opposite (additive inverse) of the subtrahend to the minuend. That is, we change the operation subtraction sign to an operation addition sign and the signs of the subtrahend polynomial to their opposites.

$$
\begin{array}{r}
4x^2 + 2x + 3 \\
\underline{\overset{+}{\ominus}\overset{-}{\oplus}3x^2 \overset{-}{\oplus} 5x \overset{+}{\ominus} 1} \\
x^2 - 3x + 4
\end{array}
$$

How can we check subtraction? (by adding the subtrahend to the minuend.)

Check:

$$
\begin{array}{r}
3x^2 + 5x - 1 \\
\underline{x^2 - 3x + 4} \\
4x^2 + 2x + 3
\end{array}
$$

Remind students of the various ways of stating subtraction problems. They can be phrased differently. Solve the following:

a) From $4x^2 + 2x + 3$, subtract $3x^2 + 5x - 1$

b) Subtract $3x^2 + 5x - 1$ from $4x^2 + 2x + 3$.

c) How much greater than $3x^2 + 5x - 1$ is $4x^2 + 2x + 3$?

d) Simplify $4x^2 + 2x + 3 - (3x^2 + 5x - 1)$.

2. Let's use the computer to subtract polynomials.

3. Load Polynomials Disk 1.

4. Type your name and press RETURN

5. Select letter G under Test Options and press RETURN.

6. Read the information about the control options.

7. You will be given 5 examples. Subtract the polynomials and then choose the letter for the matching difference.

8. Do all 5 examples this way. Note your score when you are finished.

9. Return to the Test Options and press G for 5 more problems.

10. Press CTRL and Q to quit the program.

11. Turn off the equipment and remove the disk.

Homework:

1. Subtract:

a) $$\begin{array}{r} 9x^2 - 3x - 8 \\ \underline{-\ x^2 + x - 2} \end{array}$$

b) $$\begin{array}{r} 5x + 4 \\ \underline{-\ 3x - 10} \end{array}$$

c) $$\begin{array}{r} 10x^2 + x - 2 \\ \underline{-\ 7x^2 - 7x - 9} \end{array}$$

d) $$\begin{array}{r} 5x^2 + 9x + 6 \\ \underline{-\ 3x^2 - x - 7} \end{array}$$

e) $$\begin{array}{r} 8x^2 + 4x - 3 \\ \underline{-\ x^2 - 5x - 2} \end{array}$$

2. From $6x^2 - 7x + 3$, subtract $4x^2 + 3x - 8$.

3. Subtract $x^2 + 6x - 2$ from $3x^2 - 5x + 3$.

TOPIC 11: Multiplication of a Polynomial by a Monomial

Aim: How do we multiply a polynomial by a monomial?

Performance Objectives: Students will be able to:

- recall the distributive principle
- multiply a polynomial by a monomial.

Vocabulary: Distributive property

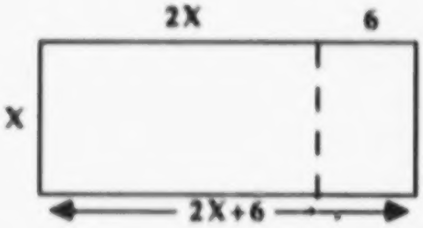
Materials:

Software: "Elementary Algebra: Polynomials" by Plato Educational Courseware
Hardware: Apple II, IIe Computers, Monitor and Disk Drive

Motivation:

What is the area of a rectangle whose length is $2x + 6$ and whose width is x?

(Hint: Try separating it into two rectangles such that the length of one rectangle is $2x$, and the length of the other is 6.)



Development:

1. We know that the distributive property states that $a(b + c) = ab + ac$
   Therefore, $x(2x + 6) = x(2x) + x(6)$
   $2x^2 + 6x$

   This can be shown geometrically by the area problem (see above).
   Area = length x width
   $= x(2x + 6) = 2x^2 + 6x$

   Use the distributive property to find the product:

   $8(2x + 3y) = 8(2x) + 8(3y) = 16x + 24y$

2. Elicit the following: To multiply a polynomial by a monomial, we use the distributive property of multiplication over addition. Each term within the parentheses must be multiplied by the monomial preceding (or following) the parentheses.
   The multiplication may also be arranged vertically:

$$\begin{array}{r} 2x - 7y \\ 5 \\ \hline 10x - 35y \end{array}$$

3. Let's use the computer to practice multiplying various polynomials by monomials.

4. Load Polynomials Disk 2

5. Type your name and press RETURN

6. From the "Test Options" display choose letter I and press RETURN.

7. Read the information about control options.

8. You will be given 6 examples to do. Multiply each monomial by each polynomial and then choose the letter for the matching product.

9. Do all 6 examples this way. Note your score when you are finished.

10. Return to the Test Options Display and press I for 6 more problems.

11. Press CTRL and Q to quit the program when you are finished.

12. Turn off the equipment and remove the disk.

Summary:

1. Find the error: $-4(x + 3) = 4x + 12$.

2. What is the procedure for multiplying a polynomial by a monomial?

3. Think of a polynomial. Multiply it by negative one. Add the result to the original polynomial. What is the sum?

Homework:

1. Find the Product:

   a) $3(x^2 + 7x)$ b) $2(5x + 1)$
   c) $4x(x^2 + 6x - 5)$ d) $7(2x^3 + x^2 + 10)$
   e) $8x(5X - 2)$ f) $-5(4x + 6)$
   g) $xy(3x^2y + 2xy^2 + 4y^3)$ h) $4(2x - 5y + 10)$
   i) $2x^2(x^3 - 9x^2 + 4x + 5)$ j) $-3x(x^2 - 2x + 9)$
   k) $(2x - 5)(-3)$

2. Find the area of a rectangle having dimensions $2x$ and $5x^2 + 7x - 1$.

3. Compute: $5a^2b^2(-5a^2 + 2ab - 4b^2)$ and check by letting $a = 2$ and $b = 3$.

TOPIC 12: Division of a Polynomial by a Monomial

Aim: How do we divide a polynomial by a monomial?

Performance Objectives: Students will be able to:

- recall the rules for division of monomials.
- divide a polynomial by a monomial.
- check the quotient.

Vocabulary: divisor, dividend, quotient

Materials:

Software: "Elementary Algebra: Polynomials" by Plato Educational Courseware
Hardware: Apple II, IIe Computers Monitor and Disk Drive

Motivation:

The total cost of a fishing trip is 4x + 4y dollars. Four boys are going on the trip. How much should each pay?

Development: (Include pivotal questions):

1. Dividing by 4 means the same as multiplying by $\frac{1}{4}$.

   Therefore,
   $$\frac{4x+4y}{4} = \frac{1}{4}(4x + 4y) = \frac{1}{4}(4x) + \frac{1}{4}(4y) = \frac{4x}{4} + \frac{4y}{4} = x + y$$

   This means that x + y can be obtained by dividing each term of 4x + 4y by 4:
   $$\frac{4x+4y}{4} = \frac{4x}{4} + \frac{4y}{4} = x + y$$

2. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. Recall the rules for dividing monomials.

   Examples:
   $$\frac{6a^4 - 10a^5}{2a^2} = \frac{6a^4}{2a^2} + \frac{-10a^5}{2a^2} = 3a^2 - 5a^3$$

   $$\frac{18x^4 + 12x^3}{6x^2} = 3x^2 + 2x$$

   To check, multiply: $6x^2(3x^2 + 2x)$ (the divisor by the quotient).

   The answer should be the dividend: $18x^4 + 12x^3$.

3. Let's use the computer to practice dividing polynomials by monomials.

4. Load Polynomials Disk 2

5. Type your name and press RETURN

6. Choose M under the test options and press RETURN.

7. Read the information about control options.

8. You will be given 6 examples to do. Divide each polynomial by the monomial and then choose the letter for the matching quotient.

9. Do all 6 examples this way. Note your score when you are finished.

10. Return to the Test Options Display and press M for 6 more examples.

11. Press CTRL and Q to quit.

12. Turn off the equipment and remove the disk.

Summary:

1. Explain the rule for dividing a polynomial by a monomial.

2. Explain the error: $\frac{18x^4 + 6}{6} = 3x^4 + 6$

3. Find the width of a rectangle whose area is $16x^2 - 12x$ and whose length is $4x$.

Homework:

Simplify and check (1 - 7):

1. $\frac{3x^2 + 12x + 21}{3}$

2. $\frac{2x^3 - 8x^2 + 32x}{2x}$

3. $\frac{8x+16}{4}$

4. $\frac{18x^3 + 30x^2 - 36x}{3x}$

5. $\frac{10x^2 + 5x}{x}$

6. $\dfrac{8x^2 - 16x + 4}{4}$

7. $\dfrac{6x^3y^2 - 12x^2y^2 + 3xy}{3xy}$

8. Find the length of a side of a square whose area is $9x^2$.

9. Find the length of a rectangle whose area is $25x^3 - 10x^2 + 5x$ and whose width is $5x$.

10. The perimeter of a square is represented by $12x - 4$. Express the length of one side of the square in terms of x.

TOPIC 13: Solving Equations $ax \pm b = c$

Aim: How do we solve equations of the type $ax + b = c$?

Performance Objectives: The students will be able to:

- recall the meaning of "solving an equation".
- isolate the variable using properties of inverses and identities.
- check the solution.
- solve equations of the type $ax + b = c$ where a, b and c are rational.

Materials:

Software: Equations published by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers Monitor and Disk Drive

Motivation:

Problem: Twice the number of people invited came to a party. Four left early and eight stayed until the end. How many people were invited?
Solution - Step One: Translate into an equation: Twice a number decreased by four equals eight.

Development:

1. Refer to the motivation. Elicit the equation: $2x - 4 = 8$. How can we find the solution?

   Method I:

   What equivalent equation would be obtained by adding the additive inverse of -4 to both members of the equation $2x - 4 = 8$?

$$\begin{array}{rcr} 2x - 4 & = & 8 \\ +4 & = & +4 \\ \hline 2x \quad & = & 12 \end{array}$$

   What equivalent equation would be obtained by applying the multiplicative inverse of 2 to both members of the equation $2x = 12$?

$$2x = 12$$

$$\left(\frac{1}{2}\right) 2x = \left(\frac{1}{2}\right) 12$$

$$x = 6 \qquad\qquad \text{Solution set} = (6)$$

2. Explain that in solving equations the variable needs to be isolated.

3. Tell the students that they will use rules to isolate the variable in solving equations of the type $ax + b = c$ on the computer.

4. Load the "Equations" disk into the disk drive and turn on the monitor and computer.

5. Type your name and press RETURN.

6. Read the introduction and press N when it asks if you want instructions.
7. Type 10 when the computer asks how many problems you would like to do.
8. Look at the equation presented on the top of the screen. You will have to pick the rule you wish to use in solving the equation. Make sure you inspect the equation closely to see what you should do to isolate the variable. You will have to simplify both sides of the equation each time you use a new rule.
9. Follow the computer's instructions entering the new left and right sides of the equation.
10. Look at the error summary after each problem to see if you are making mistakes in the procedure or computation.
11. Examine the "Total Error Analysis" after you finish all your problems.
12. Press E to exit the program.
13. Turn off the monitor and computer and remove the disk.

Summary:

1. Why is the use of just one inverse operation not always adequate to solve an equation?
2. If you changed the order in which you used the operations needed to solve an equation of the form $ax + b = c$, how would the results compare? Explain.
3. The equation $5x + 10 = 55$ has the same solution set as the equation.

   a) $x = 45$
   b) $x + 10 = 11$
   c) $5x = 65$
   d) $5x + 15 = 60$

4. Write an equation in which two inverses will be used to find the solution set.

Homework:

1. Solve:

   a) $10x - 17 = -7$
   b) $24 = 2 + 11x$
   c) $4x - 5 = -9$
   d) $-16 = 62 + 13y$
   e) $-41 = 12x - 5$
   f) $7 + \frac{1}{4}x = 15$
   g) $\frac{3x}{8} - 1 = 5$
   h) $3 + 8x = -29$

2. Find the solution set and check:

   a) $6 - 9x = -39$
   b) $24 = 2 - 11y$

3. Seven more than 3 times a number is 31. Find the number.

4. The width of a singles tennis court is 27 feet. This is one foot more than $\frac{1}{3}$ of the length. What is the length of the court?

5. Mary's age is 3 years less than two times Jim's age. How old is Jim if Mary is 21 years old?

6. Find the product and check: $(2r - 3s)(-6rs)$

7. Find the quotient and check.

$$\frac{-24uv^2 + 12u^2v}{-6uv}$$

8. Solve: $|2x + 1| = 9$

TOPIC 14: Solving Equations which Contain Variables on Both Sides

Aim: How do we solve equations which contain variables on both sides?

Performance Objectives: Students will be able to:

- state the need for removal of the variable from one side.
- use previously learned procedures for isolating the variable.
- check the results.

Vocabulary: variable, left member of an equation, right member of an equation

Materials:

Software: "Equations II" published by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers Monitor and Disk Drive

Motivation:

Do these equations have the same solution set? Explain.
a) $3x + 2x = 20$ b) $3x = 2x + 20$

Development:

1. Refer to the motivation. Review the procedure for solving equation (a). The solution $x = 4$ does not satisfy equation (b). Have students notice that the variable x appears in both the left and right members of equation (b).

2. Have students recall the use of additive and multiplicative inverses in transforming an equation such as $ax + b = c$ into simpler equivalent equations. How would these principles be helpful in obtaining a simpler equation equivalent to $3x = 2x + 20$ so that the variable appears in one member only?

   $3x = 2x + 20$
   $-2x = -2x$ (Additive inverse)
   $x = \ \ + 20$ (Solution set = 20).

   Therefore, 20 is the root of the equation $3x = 2x + 20$. Ask the students: How do we solve an equation which contains the variable on both sides of the equation?
   Elicit:
   a) Write an equivalent equation in which the variable appears only in one member.
   b) Solve the resulting equation.

3. Let's use the computer to solve equations of this type in which variables appear on both sides.

4. Load "Equations II" program.

5. Type your first and last name.

6. Read the introduction and press N for No when asked if you want instructions.

7. **Type 10 when asked how many problems you would like.**

8. **Examine the equation at the top of the screen.** (Remind the students that in order to solve the equation they must write an equivalent equation in which the variable appears only in one member.)

9. Simplify both sides of the equation after choosing each rule.

10. Enter the new left and right sides of the equation.

11. Look at your error summary after each problem.

12. Examine your "Total Error Analysis" after you finish all 10 problems.

13. Press E to exit the program.

14. Turn off the computer and monitor and remove the disk.

Summary:

1. What are the steps to be followed in solving an equation which contains variables on both sides?

2. Solve for x: $ax + b = cx$

Homework:

1. Solve and check:

    a) $6c - 20 = 2c$
    b) $8y + 11 = 7y$
    c) $7x + 8 = 4 + 3x$
    d) $-x + 3x = 6 + x$
    e) $5x - 2x = 24 - 9x$
    f) $3z + 9 = 1 - z$
    g) $5d + 11 = 8d - 7$
    h) $7x - 0.05 = 3.15 + 2x$
    i) $x + 12 + x = -14$
    J) $8y + 6y - 8 = -y - 38$

2. The sum of twice a number and 6 is the same as 11 increased by the number. Find the number.

3. Sixteen diminished by 8 times a number is the same as 5 diminished by 9 times the number. Find the number.

4. Find the quotient and check:

$$\frac{-5r^2s^2 + 30r^2s}{.1r^2s}$$

5. Solve: $7 - 2x = 5$

6. Solve for x:
    a) $ax + b = cx + d$
    b) $ax + b = c + dx$
    c) $ax + bx = c + d$

TOPIC 15: Solving Equations Containing Parentheses

Aim: How do we solve equations which contain parentheses?

Performance Objectives: Students will be able to:

- identify the parentheses as a grouping symbol.
- explain the need for removal of the parentheses.
- solve equations containing parentheses preceded by a negative sign, a positive sign or a signed number.
- check the solution.

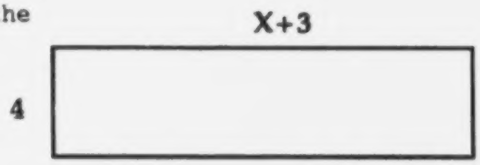
Vocabulary: distributive property

Materials:

Software: "Equations III" published by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers Monitor and Disk Drive

Development:(Include pivotal questions):

1. The area of a rectangle is 32 square inches. If the length is 4 inches and the length is x + 3, find the length in inches.



Area = (x + 3)4

Let x + 3 = length. Area = length x width.

$$\begin{array}{rcl} 32 &=& (x + 3)4 \text{ Remove parentheses using distributive law.} \\ 32 &=& 4x + 12 \\ \underline{-12} &\underline{=}& \underline{-12} \\ 20 &=& 4x \\ \left(\frac{1}{4}\right) 20 &=& \left(\frac{1}{4}\right) 4x \\ 5 &=& x \end{array}$$

Length = x + 3 = 5 + 3 = 8
Length = 8 inches

Check solution by returning to the original problem.
Area = length x width

32 = 8 x 4 = 32

2. Review, using the distributive property of multiplication over addition to simplify each of the following.

	Answers:
a) (x + 3)4	4x + 12
b) 5(x + 2)	5x + 10
c) 8(2c -1)	16c - 8
d) (x - 5)(-2)	-2x + 10

e) $3(a - 5)$	$3a - 15$
f) $-2(2a + 1)$	$-4a - 2$
g) $-2(6 - 9x)$	$-12 + 18x$

3. Solve an equation containing parentheses preceded by a negative signed number.

   Consider the equation:

$$\begin{aligned} -5(x + 2) &= 30 \qquad \text{(Use distributive property)} \\ -5x - 10 &= 30 \\ \underline{+10} &= \underline{+10} \\ -5x &= 40 \\ \left(\frac{-1}{5}\right)(-5x) &= \left(\frac{-1}{5}\right)40 \\ x &= -8 \end{aligned}$$

   Check:

$$\begin{aligned} -5(x + 2) &= 30 \\ -5(-8 + 2) &= 30 \\ -5(-6) &= 30 \\ 30 &= 30 \end{aligned}$$

4. Ask the students: How do we solve equations which contain parentheses? Elicit:
   a) Use the distributive property to remove the parentheses.
   b) Solve the resulting equation.

5. Load "Equations III" programs.

6. Type your first and last name where asked.

7. From the Main Menu select 2 for problems.

8. Press "10" for the number of problems you want.

9. Examine the problem at the top of the screen.

10. What is the first rule you must use whenever you have parentheses in an equation? (Elicit from the students that the distributive property must be applied.)

11. Press "5" to distribute over parentheses.

12. Perform the distribution.

13. Simplify the equation by combining like terms.

14. Continue to choose the correct rules and to simplify until a solution is found.

15. Look at your error summary after each problem.

16. Press RETURN for a "Total Error Analysis" after you finish all 10 problems.

17. Press "R" to return to the Main Menu.

18. Press "4" to exit the program.

19. Turn off the computer and monitor and remove the disk.

Summary:

1. Explain why $4(x + 3) = 32$ and $4x + 3 = 32$ are not equivalent equations.

2. What are the steps to be followed in solving an equation containing parentheses?

   a) Remove parentheses.
   b) Combine like terms.
   c) Apply additive inverse.
   d) Apply multiplicative inverse.
   e) Write the solution set.
   f) Check in the original equation.

3. Write an equation containing parentheses whose solution set is $\{0\}$.

4. Write an equation containing parentheses which has the empty set as solution.

Homework:

1. Solve and check:
   a) $4(x - 2) = 20$ b) $2(2s + 3) = -18$
   c) $4(2x - 1) = 20$ d) $3(x + 4) - x = 18$

2. Solve:
   a) $15 - (5x - 4) = 34$ b) $8x = 2(x + 15)$
   c) $2(3x - 5) = 2x + 6$ d) $10(x + 2) = -2(6 - 9x)$
   e) $8y - 2(-3y + 4) = -(y + 38)$

3. Five times the sum of a number and 2 is 25. Find the number.

4. Seven times the sum of a number and 4 is the same as 8 decreased by 3 times the number. Find the number.

5. Six times the sum of a number and -1 is the same as 2 more than 8 times the number.

6. If the area of a rectangle is $24r - 6r^2s$ and one side is $2r$, find the perimeter.

7. Solve: $36 - 4x = -10$

TOPIC 16: Simple Probabilities

Aim: How do we calculate simple probabilities?

Performance Objectives: Students will be able to:

- define uniform probability and random selection.
- write and apply the fundamental probability formula: $P(E) = \frac{n(E)}{n(S)}$.
- demonstrate how theoretical probability differs from empirical probability.
- provide examples requiring use of the probability formula.

Vocabulary: theoretical possibility, sample space, outcome, event, random selection, uniform probability, continuum

Materials:

Lotus 1-2-3 published by Lotus for MS-DOS microcomputers like the IBM PC. Any Lotus 1-2-3 clone will follow the keystrokes exactly. Other spreadsheets for all computers may be used with some modification but not all can create distribution tables automatically. Doing so manually for a large experiment would be unrealistic.

Motivation:

A standard deck of 52 cards is shuffled. A single card is drawn at random. What is the probability that the card drawn is: 1) a seven 2) a red three 3) any black card?

Special Preparation:

To create the spreadsheet used in this lesson, first load Lotus 1-2-3 into the computer. Globally change the column widths to 3 by typing /wgc3 and press RETURN. Move to cell A1 and type the formula for the result of a random dice roll, @INT(@RAND*6+1)+@INT(@RAND*6+1) and press RETURN. For a 100 trial experiment, copy this formula to 99 other cells by first typing /c and press RETURN. Then answer the prompt by typing a1..j10 and pressing RETURN. After a brief wait, you will see 100 simulated dice rolls (sums of two random numbers each 1,2,3,4,5, or 6). Each time you copy the original formula you will get another set of random dice rolls. Move to cell L1 and type DISTRIBUTION TABLE FOR DICE ROLLS.

Your table will consist of four columns starting in column L. Create the headings for each column as follows: In cell L3, type RESULT; in cell M3, type FREQ; in cell N3, type PERCENTAGE; in cell O3, type THEORET.

You won't be able to see the headings and results unless you first change the column widths appropriately by moving to cell L3 and typing /wc7; moving to cell M3 and typing /wc5; moving to cell N3 and typing /wc11; moving to cell O3 and typing /wc9.

Move to cell L5 and start typing the possible dice rolls vertically from 2 to 12. You can then use the data freequency capabilitites to sort a frequency table by typing/DD followed by A1..J10 when asked to specify the data range and then L5..L15 when asked for the bin range. After pressing return each time the number of instances of each sum will appear next to the values in the bin range. Then move to cell N5 and type (M5/@count(a$1..j$500)). Copy the formula into cells N6 to N15. Go to cell O5 and type each of the theoretical probabilities for each dice roll in cells O5 to O15 (i.e. 1/36, 2/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 2/26, 1/36). The computer will display the decimals for each fraction. Format the third and fourth columns of the table for percents by typing /rfp1 and press RETURN. Save on one or more data disks.

Development:

Develop the following concepts with the class.

1. Theoretical probability is used for objects known to be unbiased. Otherwise we must use empirical study.

2. Outcome is the result of some activity.
   Example: When rolling a die, the outcome is a natural number from 1 to 6 inclusive.

3. Sample space is the set of all possible outcomes of an activity.
   Example: Choose one card from a deck of 52 unmarked, well shuffled cards. The sample space is all 52 cards identified by denomination and suit.
   Note: n(S) is the number of elements in the sample space. In this case, n(S) = 52.

4. Event is a subset of the sample space (must be clearly defined).
   Example: A roll on a single toss of die produces a number greater than 3. "Event" must be distinguished from "outcome". Note: n(E) is the number of elements in the event. Note that an outcome of 4, 5 or 6 is favorable for this event. In this case, n(E) = 3.

5. Theoretical probability defined: Probability of an event (E) is the number of ways for a favorable outcome to occur divided by the number of outcomes in the sample space. $P(E) = \frac{n(E)}{n(S)}$. (Assuming all outcomes are "equally likely").
   a) Probability of an impossible event is 0.
      Example: Choose the letter x from among M,A,T,H.
   b) Probability of a certainty is 1.
      Example: Choose a consonant from T,R,H,S.
      Example: Choose a vowel from A,E,I,O,U.
   c) For any event E, P(E) is between 0 and 1 inclusive.
      Symbolically: $0 \leq P(E) \leq 1$
      In other words, P(E) falls somewhere on this continuum.

      0 .5 1
      .________________________ .
      P(E)
   d) Probability can be expressed as a ratio, fraction, decimal or percent.
   e) The sum of the probabilities of any set of events that partition the sample space for any experiment is 1.

6. Uniform probability is the expectation that all outcomes are equally likely.
   Examples:
   Flipping a coin: H or T.
   Birth of a child: boy or girl
   Tossing a die: any outcome of 1 to 6 is equally likely.

7. Random selection means that there is no tampering or interference on the part of the experimenter or anyone else.

   Load the computers with Lotus 1-2-3 and load the worksheet you created earlier. To complete the frequency table and show the experimental percentages type /dd a1..j10 and press RETURN. Then answer prompt by typing L5..L15 and press RETURN. The frequency of each type of roll will be displayed for the students as well as its relative percentage. To try again, copy the formula in cell a1 to the same range again and type the /dd commands appropriately. By copying to larger ranges, you can try experiments of various sizes.

   Question- "How do the actual percentages in the experiment compare with those we expected mathematically?"
   Elicit- They are close but almost always slightly different.

   Question- "What will happen if the number of trials is reduced?"
   Elicit- "The percentages will be even further off but still reasonably close."

   Question- "And if we increase the number of trials dramatically, what will happen?"
   Elicit- "We will get even closer to the expected percentages." You might have to show them such trials in order to elicit the last two answers.

   Question- "Can anyone summarize the difference between theoretical probability and empirical results in your own words?"
   Elicit- "Theoretical probability is what mathematics tells you to expect given a large number of trials. However, in reality it is only a guide. The actual results vary.

   Question- "Don't answer but think about this. Is this a situation where luck plays a role in mathematics?"

Materials: deck of cards, dice, coins

Applications:

1. Consider a spinner (from a board game) with 10 congruent regions. Unbiased, theoretical probability applies. The arrow must point only to one region (if on line, do over).
   a) What is n(S)?
   b) Calculate P(7)
   c) Event: Any even number greater than 6. Calculate P(E).
   d) Event: Any number less than 5. Calculate P(E).
   e) Event: Any odd number. Calculate P(E).

2. A card is chosen from a well shuffled deck. Find:
   P (any ace)
   P (red Jack)
   P (any card except 7)

3. A fair die is rolled. Find:
   P (even #)   P (# >1)
   P (prime #)   P (5 or even #)

4. Some Easter eggs are in a basket, colored as follows: 4 green, 2 blue, 1 yellow. An egg is randomly selected.

Find:

P (green)	P (not blue)
P (not yellow)	P (white)
P (yellow)	P (not orange)

Summary:

1. What is theoretical probability?

2. State the fundamental probability formula.

3. What are the restrictions on the number which expresses a probability?

Homework:

1. A bag contains 3 red marbles and 2 green marbles, indistinguishable except for color.

   a) If a single marble is selected at random, what is the probability that it will be:
   1) red 2) green?

   b) If a marble is selected, returned and again a marble is selected, we may show the sample space as:

		C	D
2nd Selection	Green	o	o
	Red	o	o
		A	B
		Red	Green
		1st Selection	

   (1) What is the probability of each pair: A,B,C,D?
   (2) Viewed this way, are the 4 outcomes equally likely?
   (3) How can we show the sample space to reflect uniform probability?

   c) Suppose now that 2 marbles are selected at the same time. We may picture its sample space also.

G 2	o	o	o	o	
G 1	o	o	o		o
R 3	o	o		o	o
R 2	o		o	o	o
R 1		o	o	o	o
	R 1	R 2	R 3	G 1	G 2

(1) Why are subscripts shown?
(2) Why are the diagonal dots omitted?
(3) How many possible outcomes are there?
(4) What is the probability that
    a) both marbles are red?
    b) both marbles are green?
    c) one marble is red and the other is green?

(5) If we make a 2 marble selection one at a time, without returning, what is the probability that
    a) The 1st marble is red and the 2nd green?
    b) The 1st marble is green and the 2nd red?
    c) The 1st marble is red or the 2nd is green?

2. a) Find these probabilities:
    1) $P(r = g)$
    2) $P(r > g)$
    3) $P(r > g)$
    4) $P(r = 2g)$
    5) $P(r > 1 \wedge g > 1)$
    6) $P(r > 1 \vee g > 1)$
   b) Picture its sample space in a manner that reflects uniform probability.

3. A card is chosen at random from a standard deck of 52 cards. What is the probability that this card is:
   a) a picture card.
   b) an ace and red.
   c) an ace or red.
   d) a picture card or red.
   e) not a picture card.
   f) neither a picture card nor red.

4. A penny is tossed and a die is rolled. Suppose that the head outcome stands for 2 while the tail outcome stands for 3. The outcomes of both are added:

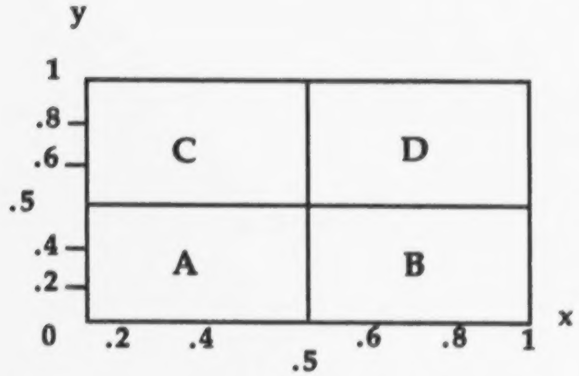
   a) Picture the sample space for this experiment.
   b) What is the probability that the sum will be:

    1) 1
    2) 2
    3) 3
    4) 4
    5) 5
    6) 6
    7) 7
    8) 8
    9) 9
    10) 10
    11) more than 5
    12) more than 2

5. An English test consists of 25 multiple-choice questions with 5 alternates for each question. For each incorrect response there is a penalty of 1 point. If the question is not answered there is no credit and no penalty.
   a) Is the penalty justified? Why?
   b) Under what circumstance is it wise to guess?
   c) Suppose the test consisted of only 20 multiple choice questions? Is the 1 point penalty justified? Why?

6. A computer is used to generate random numbers in the interval from 0 to 1. Two such numbers are used to obtain the x and y coordinates for locating a point. What is the probability that a pair of random numbers will locate a point in region:

a) A c) C
b) B d) D



7. How can the computer be used to find the area of region A in exercise 4? If you have access to a computer, check to see how close your answer agrees with P(A).

8. Challenge problem: Four men go into a night club and check their hats. The hat check person is fired from her job during the evening and absconds with the checks. What is the probability that exactly three men get their correct hats back.

9. Challenge problem: TRISKAIDEKAPHOBIA - fear of the number 13. FRIDAY THE THIRTEENTH day of the month occurs 48 times every 28 years. What is the probability that any given Friday falls on the 13th?

Two acceptable answers:

using 365 day year $\frac{48}{1460}$

using 365.25 day year $\frac{48}{1461}$

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7	6	9	5	6	7	7	4	3	9
9	4	10	7	8	11	10	6	11	5
7	3	3	7	7	6	4	9	4	3
9	7	2	3	7	7	6	7	7	11
7	11	7	11	5	5	5	4	8	4
5	8	12	10	12	8	3	6	8	8
8	7	10	9	9	6	8	7	4	7
6	12	7	5	7	10	8	7	7	3
8	10	8	6	9	8	7	5	3	11
12	8	3	5	6	7	9	8	4	9
6	9	6	5	10	7	7	9	8	10
7	5	7	3	6	7	4	5	3	11
5	9	5	6	12	5	3	9	8	11
11	12	2	10	10	8	9	3	9	9
10	8	8	6	2	5	8	6	10	5
2	8	9	3	4	6	5	6	7	6
9	4	5	6	7	9	6	7	3	6
4	8	8	12	9	7	5	4	8	10
2	7	6	9	6	9	3	7	5	6
4	9	6	7	9	7	6	9	4	7

Distribution Table for Dice Rolls

RESULT	FREQ.	PERCENTAGE	THEORETICAL
2	5	2.5%	2.8%
3	16	8.0%	5.6%
4	15	7.5%	8.3%
5	20	10.0%	11.1%
6	27	13.5%	13.9%
7	37	18.5%	16.7%
8	24	12.0%	13.9%
9	27	13.5%	11.1%
10	13	6.5%	8.3%
11	9	4.5%	5.6%
12	7	3.5%	2.8%
	n = 200		

TOPIC 17: The Counting Principle

Aim: How do we use the counting principle to determine n(S), the size of sample spaces?

Performance Objectives: Students will be able to:

- state the definition of the counting principle.
- apply the counting principle to problems involving menu selections, rolls of dice, urn problems. (calculator or computer applications).
- use tree diagrams to illustrate the counting principle.

Vocabulary: counting principle, tree diagram

Materials:

Lotus 1-2-3 for MS-DOS computers was used here. However, any spreadsheet will do for any computer. The point is to give students the opportunity to work with the rather large results generated using the counting principle on real life problems. You also want to give the students an opportunity to experiment.

Special Preparation:

Load your spreadsheet into the computer and make two changes in the format. First change the column width to 20 so that large answers may be accomodated by typing /wgc20 and press RETURN. Then change the format of the numbers by typing /wgf followed by the comma (i.e. ,) and then answer the prompt with a zero. This will cause all data to be displayed as a whole number with commas inserted between groups of three digits for ease of reading. Note that the largest integer value that can be displayed is dependent on your hardware and the software. The Lotus 1-2-3 clone used in this sequence of examples switched automatically to scientific notation for numbers larger than 13 digits. This allows for numbers less than 10 trillion in magnitude. Save multiple copies of this worksheet for classroom use.

Motivation:

Today the cafeteria offers:

MAIN COURSE	VEGETABLE
stew	beets
or	corn
ravioli	mushrooms

If you must choose one vegetable with your main course, list all of the possible lunch combinations.

Development:

1. Discuss motivation problem. Show how a tree may be used to ensure that all six possibilities are considered.

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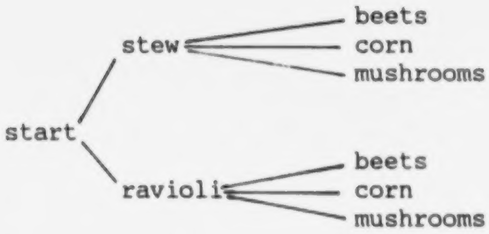


Elicit that the number of possible combinations may be calculated by multiplying the number of main course offerings by the number of vegetable offerings. State the counting principle: If one activity can occur in m ways and following this a second activity can occur in n ways, then the two activities can occur in the order given in mn ways.

2. Generalize the above example by asking: Suppose you are also given a choice of five desserts; how many complete lunch-combinations are now possible?
   Elicit that the tree could be extended with five new branches from each old branch tip; thus, the answer is 2x3x5 = 30.
   Generalize the counting to apply to longer sequences of activities.

3. Load your computers with Lotus 1-2-3 and retrieve the specially designed worksheet for each computer.. Have students consider the following and use the computer for the calculations as you go along.
   Note: Students should have been previously instructed during the year on how to enter simple formulae into a computer (e.g.,* for multiplication, / for division, ^ for exponents, the use of parentheses, etc.)
   Consider: How many different license plates can be formed using the 26 letters of the alphabet and the ten digits if they consist of 3 letters followed by 3 numbers?
   Elicit: 26 x 26 x 26 x 10 x 10 x 10 and after placing the formula on the board have the students enter it into their spreadsheets to arrive at 17,576,000.
   Consider: That's an adequate number given the number of cars in New York State (that's how the new license plates look, although there are instances of 4 numbers across). If however, we needed a larger number, how could we enlarge the license plate?
   Elicit: Add more numbers or letters.

4. Show the class how an additional digit increases the number to 175,760,000 which is far too high and the extra digit increases the cost of the plate. The same would be true for adding an extra letter however then you end up with 456,976,000 which is even more wasteful.

5. Ask: "Would a license plate made of five letters be enough for the auto population of N.Y. State?
   Elicit: When I try it on my computer I get 26^5 or 11,881,376. Is that enough?
   Refer students to latest edition of almanac or other reference material to find answer.
   Students can also try 4 letters and 2 numbers, 5 letters and 1 number, etc.

Note: Remind students that the existence of vanity plates distorts the actual results plus certain combinations of letters are either reserved or forbidden thus distorting the above figures even more.

6. Consider: Many New Yorkers play Lotto each week. Most states now have some form of this game available. How many possibilities are there for choosing 6 numbers out of the first 40 whole numbers?

   Elicit: 40 x 39 x 38 x 37 x 36 x 35 which comes out to 2,763,633,600 on their computers. Those students who ask why 40 x 40 x 40 x 40 x 40 x 40 is not right should be reminded that each number on the game form for a particular game can be chosen only once so you have fewer possibilities for each successive choice.
   You might take a minute or two to investigate the intelligence of gambling when the odds of success are on the order of 1 in 3 billion for the first prize when such a prize is generally worth only a few million dollars.

7. Students can try other possible amounts of Lotto numbers and can see what happens if you change the 40 to some other number.

   Elicit: If the numbers or letters cannot be reused then the final result must be somewhat smaller.

Applications:

1. A menu offers 5 main courses, 3 beverages and 7 desserts. How many different meals can be ordered including:
   a) 1 main course, 1 beverage, 1 dessert?
   b) 1 main course, 1 beverage, no dessert?
   c) dessert and beverage only?

2. A girl has 4 sweaters and 5 skirts. How many different outfits (sweater and skirt) are possible? Assume no color clashes.

3. A student has a true-false test with ten questions. How many different answer keys are possible? (Hint: If you have trouble with this, try it first with 3 questions as an introduction.)

4. Baskin-Robbins offers 31 flavors of ice cream. How many different double scoops can be made if the two flavors may not be the same?

Summary:

1. State the counting principle.

2. How many 7-digit phone numbers are possible (assuming that the number cannot start with zero)? Does this explain why some metropolitan areas require several area codes?

Homework:

1. How many two-letter "words" are there that begin with a vowel and end with a consonant?

2. How many two-letter words are there with no repeated letter?

3. How many two-digit numbers have a prime for the tens place?

4. How many two-digit numbers have an odd number for the tens digit and an even number for the units digit?

5. How many three-digit numbers have an odd digit in the hundreds place?

6. How many three-digit numbers have a prime in the hundreds place, an odd number in the tens place and an even number in the units place?

7. A man has 3 ways of traveling from A to B and 5 ways of going from B to C. How many ways does he have to make the trip from A to C by way of B? List all these ways.

8. A class consists of 12 boys and 13 girls. How many President, Vice-President election pairs are possible if the pair must be of opposite sex? If the Vice-President must be a girl?

TOPIC 18: Compound Events

Aim: How do we calculate the probabilities of compound events?

Performance Objectives: Students will be able to:

use the computer to verify the counting principle when independent compound events are involved

Vocabulary: independent events, dependent events, ordered pairs of elements in sample space, ordered triples of elements in sample space, ordered n - triples of elements in sample space

Materials:

The software used was a Lotus 1-2-3 clone available for all MS-DOS compatible microcomputers. However, any spreadsheet for any micro that has the facility to group data and display the results in columnar form will suffice. Lotus 1-2-3 is published by Lotus.

Special Preparation:

Create a spreadsheet to simulate the tossing of 3 coins using the random number function. If you let 0 stand for heads and 1 stand for tails, you can easily tell how many heads have been thrown and how many tails by adding the results of 3 random integers using 0 and 1. A sum of 0 means 3 heads were thrown. A sum of 1 means 2 heads. A sum of 2 means 1 head and a sum of 3 means no heads were thrown. Use the concept of complement to find the corresponding number of tails in each case.

After loading in the spreadsheet program, set up your random coin toss by writing the formula @INT(@RAND*2) in cell A1. Then copy it into as many cells as possible given memory restrictions of particular machine. I was able to copy it into the first 240 rows of the first 3 columns by typing /C followed by A1..C240 and press RETURN. This simulates the experiment 240 times. 1000 times would obviously yield more accurate results. In column D, type the formula @SUM(A1..C1) and then copy it into cells D2 through D 240. Starting in cell F1 and going down the column, type the numbers 0,1,2 and 3. You can then use the data frequency capabilities to sort a frequency table by typing /DD followed by D1..D240 when asked to specify the data range and then F1..F4 when asked for the bin range. After pressing return each time, the number of instances of each sum will appear next to the values in the bin range. For example, if there are 91 ones then that means that of the 240 throws there were 91 instances of throwing 2 heads and 1 tail. To repeat the experiment on the spreadsheet, type /DD and press the enter key twice. Run the experiment in class a few times and keep track of the results manually or enter them into a blank section of the spreadsheet for later summation and analysis. Make sure you save one or more copies on your data disk for later use.

Development:

Explain how you are going to test the multiplication rule for compound events by experimentation on a set of random data. You are simulating the throwing of 3 coins using numbers to stand for heads (0) and tails (1). Show how the number of heads or tails can be calculated via the sum. Run the experiment one or more times until you have made at least 1000 tosses of the 3 coins and keep a frequency table of the results. Better yet would be to have a few groups of students do the experiments at separate terminals so that a few thousand tosses are made for more accurate results. Then perform Application #2 using both the sample space roster and the multiplication rule for compound events. See how the results compare with the experiment.

Question--Do the results of the experiment prove or disprove the theory especially when you consider the probability factor involved?
Elicit--Yes although a number of trials are necessary to see the pattern clearly.

Question--If we wanted to know the probability of throwing at least 2 heads experimentally, how would we do it?
Elicit--Add up the number of instances of 2 heads (sum of 1) and 3 heads (sum of 0) and divide by the total number of tosses (each toss consisting of 3 coins).

Question--If we were to try the experiment with 1 coin and 1 die, how could we set up the conditions of the spreadsheet? This could be set up as an extra credit example for the average class.
Elicit--One possibility is to let the numbers 1 through 6 stand for the spots on the die. Then let the number 7 stand for heads and 13 stand for tails. The sample space roster consists of 12 distinct possibilities and the sum of any two numbers (1-6) and (7,13) result in 12 distinct values. The same type of experiment can be run.

Applications:

1. If a coin is tossed and a die is rolled, what is P(H 5)?

2. If 3 coins are tossed simultaneously, are the tosses dependent or independent? (independent)

Calculate:	Answers:
P (all heads)	$(\frac{1}{8})$
P (at least 2T)	$(\frac{4}{8})$
P (exactly 2H)	$(\frac{3}{8})$
P (at least 1T)	$(\frac{7}{8})$
P (not all heads)	$(1 - \frac{1}{8} = \frac{7}{8})$

3. Calculate the probability of at least 1 boy in a family with 3 children. (Answer $\frac{7}{8}$)

4. If all questions are answered by guessing, compare the probability of answering 3 questions on a true-false test correctly with the probability of answering 3 questions on a matching test (each test has 3 questions).

Answer: On T-F, P(all correct) = $\frac{1}{8}$

On matching, P(all correct) = $\frac{1}{6}$

Which is easier, T-F or matching? (See homework problem #5.)

Summary:

1. How do we evaluate probabilities of compound events?
   Method 1:
   a) List the sample space by a tree diagram or ordered elements. Count n(S).
   b) Count n(E), the number of elements that satisfy the given conditions.
   c) Use formula: $P(E) = \frac{n(E)}{n(S)}$

   Method 2: Multiply the probabilities of the separate independent events.

2. The probability that it will rain tomorrow is $\frac{2}{5}$. The probability that you will have spinach for dinner tonight is $\frac{1}{4}$. What is the probability that you will have spinach tonight and it will not rain tommorrow?

Homework:

1. A coin is tossed 4 times. What is the probability that:
   a) 4 heads will turn up?
   b) 3 heads and 1 tail will turn up?
   c) 2 or more heads will turn up?
   d) 2 heads and 2 tails will turn up?

   (A tree diagram might help.)

2. 6 coins are tossed. What is the probability that there will be 3 heads and 3 tails?

3. A red die and a green die are rolled. What is the probability that the red die will show an odd number of dots while the green die will show fewer than 5 dots? Show a tree diagram for this experiment. Using your tree, do you get the same answer as using the counting principle?

4. There are 5 spots on the gym floor, 1,2,3,4,5. How many different ways could Amy, Bess and Caroline occupy these spots?

5. Calculate the probabilities of guessing (randomly) all answers correct in a:
   a) 6 question true-false quiz. (answer: $\frac{1}{64}$)
   b) 6 question matching test. (answer: $\frac{1}{720}$)

(Assume that you know none of the answers at the start of the test.)

6. The probability that George will fail French is .2 and the probability that he will fail math is .25; find the probability that he will pass both courses.

TOPIC 19: Introduction to Statistics

Aim: What is meant by statistics and data in mathematics?

Performance Objectives: Students will be able to:

- define statistics as a discipline dealing with the collection, organization, and interpretation of collected information called data
- examine sampling techniques and principles used by statisticians
- explain the role of database programs in helping people keep statistics

Vocabulary: statistics, data, sample, random, population, inference, database program

Materials:

Hardware: 16-32 Apple IIc or IIe computers, each with at least one disk drive.
Software: The "Apple at Work Sampler" program.

Motivation:

1. Instruct students to work with the Apple at Work diskette (described in Appendix A) according to the instructions given in Appendix B.

Development:

1. What type of computer program were you working with? What was it used to do?
   a. Students may observe that a database program is used to organize data.

2. What were some of the things you were told about the celebrities?
   a. List data items by grouping them into numerical data (e.g., year of birth) and alphabetical data (e.g., name).
   b. Elicit the difference between numerical data and alphabetical data (i.e., that numerical data can be manipulated mathematically).
   c. Explain that both forms of data are examples of statistics kept on people.
   d. Ask students to define statistics.

3. Did the database give you information about every single person in the United States? Then why is this called a sample?
   a. Elicit the fact that a sample involves choosing only a part of the total number of people in the population.
   b. Explain that the data obtained from the people in the sample is then used to make judgements about many more people in the population.

4. Once the newspaper columnist selected the people that she would like in her sample, how do you think she obtained all the information that she did about them?
   a. Ask "What is a survey?"

5. Let's talk about occupations. Can we say that every single person in the United States is an actor or a comedian?

Then why is this an unfair sample?

a. Students may observe that the celebrities in the sample are different from "regular" people in the rest of the population.

b. Ask "Can data about celebrities be used to draw conclusions about common people like you and me?"
   Explain that the fairest kind of sample is called a "Random" sample.

c. Ask "What does it mean when we say that people are picked at 'random'?"

d. Define a random sample as one where everyone in the population has an equal chance of being picked.

Application:

1. Describe the following statistics to students and have them explain why they are fair or unfair.
   a. The incomes of 10 people picked by a computer according to their social security number. (fair)
   b. The incomes of all subscribers to the Wall Street Journal. (unfair)
   c. The heights of all the people in this class. (unfair)
   d. The heights of 1000 high school students throughout the United States. (fair)

Summary:

1. How is a sample connected to its population?

2. How is a random sample different from an unfair sample?

3. How would we define statistics?

4. How do database programs help us to study statistics?

Homework Assignments:

1. Bring in a newspaper or magazine item which includes statistics. (What are they? How were they obtained? How are they interpreted?)

2. What numerical and/or alphabetical data could we compile about: (a) our classmates, (b) U.S. colleges, (c) cars

Appendix A

(For the teacher)

The Apple at Work is a program which introduces computer users to the three major types of software used in business: (a) word processors, (b) databases, and (c) spreadsheets. The starting procedure involves simply sliding the disk into the disk drive and turning on the computer. It takes about 15 minutes to run through the database segment of the program which is used in the motivation of this lesson.

Appendix B

(Directions for the students on operating the Apple at Work.)

1. Hold the disk by the label so that the side labeled "The Apple at Work" faces up.

2. Slide the disk into the disk drive labeled "Drive 1" and close the door.

3. Turn on the monitor then turn on the computer from the rear end of left-hand side. (You will see the red light on the disk drive turn on.)

4. When a list called the "APPLE AT WORK MENU" appears on the screen, pick choice #3 (DataBase Programs) and press return.

5. When you have finished the lesson, simply remove the disks and turn off the computer and the monitor.

TOPIC 20: Statistical Data, Tables and Graphs

Aim: How do we organize data into graphs and histograms?

Performance Objectives: Students will be able to:

- present, analyze and interpret data in picture graphs, bar graphs, and histograms.
- construct graphs to show organized data.
- construct graphs using a microcomputer to show organized data.

Vocabulary: picture graph, bar graph, line graph, circle graph, histogram, interval, cumulative frequency, frequency

Materials:

- computer lab
- electronic spreadsheet that does manipulation of data and presentation of quality graphics displays

Motivation:

Take an integrated spreadsheet program such as Lotus 1-2-3 that does both spreadsheet manipulation of data and presentation of quality graphics displays. Enter some data, either from a previous related lesson or new data. Use the program to create a bar graph and a circle graph. Then, during class, display both graphs and the data from which they arose. Manipulate one piece of data and see how the graphs change. Question - "Why is this (a graph) better than showing the raw data?"

Development:

1. Discuss motivation. Students should observe that graphs are more interesting and draw attention visually to the most dramatic statistical figures. Have students mention other types of graphs they have seen (i.e. line, pictograph, histogram).

2. Briefly, mention the strengths of each type:
   a. In a pictograph, an appealing symbol is used to represent a given quantity of something. It is obvious at a glance what topic the graph discusses.
   b. A circle graph dramatically shows the distribution of values among parts of a whole. (These are frequently used to illustrate budgets.) You might, at this point, show an exploded circle graph on the computer screen, that really emphasizes a particular piece of information.
   c. A line graph is particularly useful to reveal changes (or trends) over time. (Hospital temperature chart, company sales performance are good illustrations.) (Lotus 1-2-3 can be used to show a line graph.)
   d. A bar graph facilitates comparison of any two values displayed.
   e. A histogram is a vertical bar graph which illustrates frequencies for intervals of continuous data. It has no space between the bars; the bars are placed together to emphasize the idea of continuity: one interval begins where the previous one ends.

3. Give the students a frequency table for an interesting topic such as the one for a previous lesson on the number of TV-viewing hours per family shown as follows:

Number of Hrs. in 10 wk. period	Frequency
100 - 124	4
125 - 149	9
150 - 174	46
175 - 199	13
200 - 224	7
225 - 249	1

4. Have students create a histogram, by hand, using graph paper and a straight edge. Ask students to respond to the following questions:
   a. What was the behavior of a typical family?
   b. What drew the fewest responses?
   c. How many families reported watching fewer than 200 hours?
   d. What percent of the families surveyed lies in the interval that drew the most responses?

Applications:

1. Construct a histogram based on the student heights problem of the previous lesson.

2. Construct histograms based on the applications problems of that lesson.

3. Load any of a number of simple graphing utilities and give the students the opportunity to create their own graphs. This might take part of a second period to complete, but it would be time well spent since most graphs tend to be computer generated these days. Special attention should be paid to the following:

   a. Most graphing software works in two modes. The vertical or frequency axis is numbered automatically (based on the data) or can be dealt with manually. Students should be encouraged to experiment with both methods. In the manual mode, students will see that if the axis is numbered without any thought as to the data itself, the graph will be almost meaningless.
   b. If the graphing utility allows the raw data itself to be used, this becomes easier; however students should experiment with different size intervals and different numbers of intervals. Students will discover that too many or too few intervals also distort the resulting graph, again making it meaningless.

Note: Lotus 1-2-3 could be used for the above hands-on work but would probably be too difficult. For demonstration purposes (outlined in the Development) it would serve nicely.

Summary:

1. Why are the bars placed against each other in a histogram?

2. What are the special advantages of a histogram (compared to other types of graphs)?

3. Identify another situation in which you might wish to use a histogram to communicate information.

4. Suppose you collected data about what type of pet each student in the school owned. Would you use a histogram to display this information? Why or why not?

Homework:

1. Construct histograms based on the projects described in the homework in the last lesson.

2. Bring in a pictograph, line graph, circle graph or bargraph taken from a newspaper or magazine. Write several conclusions about the data presented. Why was this form of graph chosen, rather than a histogram?

TOPIC 21: Mean, Median, Mode

Aim: What do we mean by measures of central tendency?

Performance Objectives: Students will be able to:

- list the three ways used to express the center of a group of data
- calculate and interpret the measures of central tendency using numbers generated by the program

Materials Needed:

Hardware: 16 to 32 Apple IIc or Apple IIe computers. Each should be equipped with at least 1 disk drive.
Software: The Basic Math Competency Skill Building Series on Mean, Median, and Mode. (Written by Conlan & Gilmore. Published by Educational Activities, Inc. of Freeport, N.Y., 1984.)

Motivation:

Helen's test scores so far this term have been 82, 78, 83, 81, 75, 90, and 83. What single grade would you use to represent her test performance in this class?

Development:

1. Discuss the motivation.
   a. Students will probably be familiar with the process of "finding the average" by adding all the scores and dividing by the number of tests.

2. Instruct students to run the Basic Math Skill Competency Program (described in Appendix A) in accordance with the instructions given in Appendix B.

3. Discuss the three ways used to express the center of a group of data. Ask:
   a. "The program talked about the three ways of finding the center of a group of data. What are they?"
   b. "How do we get the mean?"
   c. "What is meant by the median?" "The mode?"

4. Elicit the following definitions and illustrate each using the data given for Helen in the motivation:

   a. Mean - the average of a group of scores (the total of the scores divided by the number of scores).

      Mean = (82+78+83+81+75+90+83)/7
      Mean = 572/7 = 81 5/7 or approximately 81.7142

   Median - the number that appears in the middle of the group (put all the numbers in order and pick the middle number).

      75,78,81,$\hat{82}$,83,83,90

      Median

Mode - the number that appears most frequently.

$75,78,81,82,\underset{\frown}{83},\underset{\frown}{83},90$

Mode

Ask: "How do we represent the median of an even number of items such as 3, 5, 6, 7, 7, 8?"

$3,\ 5,\ \underset{\frown}{6},\ \underset{\frown}{7},\ 7,\ 8$

$$\begin{aligned}\text{Median} &= (6 + 7)/\ 2 \\ &= 6.5\end{aligned}$$

5. Five people have the following annual salaries: $20,000; $22,000; $25,000; $38,000; and $63,000. Find the mean, median, and mode.

   Is the mean ($33,600) really a good number to use as typical in representing these five salaries? Discuss the shortcoming of using the mean when there are numbers which are quite different from the other numbers. The median ($25,000) is a better expression of a typical annual salary for this group. The set has no mode.

6. Discuss the advantage of knowing which number occurs most frequently (the mode).
   This number can be used by a clothing store, for example, in ordering sizes.

Applications:

1. If the sum of several items is 816 and the mean is 48, how many items are in the data?

2. In the motivation problem, what mark does Helen need on the eighth test to bring her average up to an 83?

3. For which set of numbers are the mean, mode and median all equal?
   {2,2,5}, {2,5,5}, {2,2,5,5}, {2,3,4,4,7}

4. How would the mean be affected if each term in a distribution was doubled? ...was increased by 3?

Summary:

1. Name three ways of expressing the center of a group of data and explain the difference between them.

2. Give an example of a situation in which you would be most interested in using the mode?
   -- Repeat question for mean and median.

3. The average (mean) height of a group of seven students is 63". Ann, who is 65" tall, joins the group. What is the new mean height?

Homework:

1. Calculate the mean, median, and mode for each set of data:
    a) 19,20,17,24,29
    b) 47,85,300,85,53,47
    c) 42,43,44,43,40,49
    d) -7,-3,-1,2,-1,-3,0
    e) 18.7,19.4,17.25,16.8,18.3,19.2,17.1,19.4
    f) 2 1/2, 4 1/2, 9 3/4, 5, 2 1/2

Appendix A (For the teacher)

The diskette for this lesson is the Basic Math Competency Skill Building Series on Graphs and Mean, Median, and Mode. This is a tutorial diskette designed to introduce students to the aforementioned topics.

The program generates groups of scores for which the students are then required to total and later divide by the number of scores in order to compute the mean. The program then sorts these numbers in ascending order to permit the student to pick the median and the mean.

Apart from requiring students to add and divide numbers using pencil and paper, an advantageous feature of this program is that the groups of scores are produced by a random generator so that it can be used repeatedly.

It is strongly suggested that the teacher become familiar with the directions given to students in both Appendix B and the program itself before attempting to use the software in the classroom.

Appendix B

1. Put the diskette into disk drive A; then turn on both the monitor and the computer.

2. When the main menu appears on the screen, you will be asked to pick between:
    (1) Graphs
    (2) Mean, Median, Mode
    (3) Exiting the system

3. Pick number 2.

4. Another menu will then appear that will ask you to pick between:
    (A) Sample
    (B) Quiz
    (C) Return to Menu

5. Pick letter A.

6. Follow the directions given to you by the computer until you feel you can answer the questions given to you in the WARMUP.

TOPIC 22: Grouped Data

Aim: How do we use grouped data to determine measures of central tendency?

Performance Objectives: Students will be able to:

- . construct frequency tables with appropriate intervals with respect to quantity and size.
- . determine the interval containing the median and the modal interval from frequency distribution tables.
- . use the computer to handle the grouping of large quantities of data faster and more accurately than tallying by hand.

Vocabulary: grouped data, frequency table, modal, interval

Materials:

The software used was a LOTUS 1-2-3 clone available for all MS-DOS compatible microcomputers, however many advanced spreadsheets have the facility to group data and display the result in columnar form. Lotus 1-2-3 is published by Lotus.

In addition, the teacher may wish to duplicate or develop transparencies for the attached data sheet and frequency table examples.

Motivation:

These are the ages of people living in a particular apartment building:
36,22,3,41,32,10,68,5,11,39,22,35,12,5,9,57,37,59,42,33

1. Calculate the values of the mean, mode and median.

2. Make a frequency table with ten tallies in each interval (i.e., first interval is 0-9, next is 10-19...)

Special Preparation:

Enter 60 pieces of data to be used for this demonstration into the first 20 rows of columns A, B, and C of your spreadsheet. The attached example works well, but your own set of student test scores can be adapted with minimal difficulty. Print out the new data by typing /PPR and then define the range as A1..C20. After making sure the printer is connected properly hit the G or Go command to obtain your raw data printout. Duplicate as necessary. The best way to go about doing this is to demonstrate the computer's grouping abilities once and then hand out the second worksheet or display it on an overhead projector. In any case, the four examples should be done in advance and saved as separate files. To group the data go to cell E1 and in a column you would type the upper bound of each interval used in the particular example. For example, to produce the data necessary to generate table #3 on page 99 type 49, 59, 69, 79, 89 and 99 in column E in the first 6 rows. Then type the following keystrokes: /DD followed by A1..C21 to define the range of data being used. After pressing RETURN, define the range of groups by typing E1..E6 and press RETURN.

Immediately the numbers 6,1,4,11,25, and 13 will appear next to the numbers in column E. Each number is the frequency obtained by checking the data for all numbers less than or equal to the given value in E but greater than the preceding value. In other words, there are 6 values less than or equal to 49; one value less than or equal to 59 but greater than 49; 4 values less than 69 but greater than 59 and so on. The corresponding frequency tables can be constructed in a separate portion of the worksheet or by hand. Save each one separately or create the group ranges in separate portions of one large worksheet and save the whole thing.

Development:

1. Discuss motivation problem. The mean is 28.9; the median is 32.5; the modes are 5 and 22 (the set is bimodal).

Age	Frequency
0-9	4
10-19	3
20-29	2
30-39	6
40-49	2
50-59	2
60-69	1

2. Ask: If you had been given the frequency table rather than the data, could you have calculated the values of mean, median and mode? (No)
   Ask: Could you have predicted which intervals would contain the modes? (No)

   Define: The modal interval is the interval with the greatest frequency of data.
   Ask: Which is the modal interval for this data? (30-39)
   Note: In this case, the modal interval does not contain either mode.
   Ask: Given only the frequency table, could you have predicted which interval would contain the mean or the median?
   Mean: No. Consider the effect of replacing the values 32,41,42 of the original set with 39,49,49; the frequency table would be unchanged but the mean would move to a different interval.
   Median: Yes! The interval can be predicted using the following strategy: since there are twenty numbers in the set, the median must be the average of the 10th and 11th data elements (in size-order arrangement); since both of these lie in the 30-39 interval, the median will lie in this interval also. This strategy will always work when the number of elements is odd.

3. Boot up the raw data worksheet on one or more computers in the lab. A large screen TV is preferable when one computer is being used for a demo. Display the raw data and ask the following:

   Question - What level of grades did most of the students get?
   Question - How many scored higher than 90?
   Question - If 70 miss passing, how many passed?
   Question - Are the scores evenly distributed or are they skewed?

Elicit - A frequency table similar to the one used previously would be helpful in answering those questions.
Question - Wouldn't it take quite a while to tally up everything?
Elicit - What about using the computer to do it for us?

4. At this point, go through the explanation described in the SPECIAL PREPARATION section on page 95 and show how the computer will tally the scores into whatever groups or intervals we wish. Depending on time, do this demonstration one or more times and then hand out the rexo of frequency values or display it on the overhead projector. The spreadsheet itself can display it but that isn't really necessary.

5. Show students frequency table #3 and ask them the same questions as before. This time the answers are more obvious.

6. Also ask them to find the modal interval and the interval containing the median.

7. Now ask them to look at the other 3 frequency tables and ask the following:

   For table #2:
   Question - Can you quickly get a full picture of the tendency of the data with this table?
   Elicit - No, there are too many intervals. It's just a little better than taking all the data and sorting it.

   For table #3:
   Ask the same question.
   Elicit - You can get a picture quickly but it doesn't tell you much other than that a lot more people did well than poorly.
   Note - For all you know, all 49 people scored 70 and the table would look the same so you really would use only 2 intervals in a pass-fail situation.

   For table #1:
   Ask the same question.
   Elicit - It sort of gives you the picture but it distorts it so that you have to look at it more carefully. Also, the modal interval becomes meaningless because some intervals are smaller or larger than others.

8. Ask class to summarize their findings.
   Elicit - A frequency table should use a reasonable number of intervals, all of the same size.

9. Tell class that usually 4 to 7 intervals is a good number unless you have a tremendous amount of data (i.e. all the reading scores in the entire school).

SPECIAL NOTE TO TEACHERS: THIS TECHNIQUE CAN BE USED WITH YOUR OWN TEST SCORES TO DEVELOP FREQUENCY TABLES AFTER EACH TEST, SOMETHING MANY TEACHERS IN SECONDARY GRADES DO BY HAND FOR 4 OR 5 CLASSES (WELL OVER 100 PIECES OF DATA).

Applications:

Using frequency tables made for problems considered previously, ask students to identify the modal interval and the interval containing the median.

Summary:

1. Will the mode lie in the modal interval? Explain.
2. Is it always possible to identify the interval containing the median from a frequency table? Explain.

Homework:

1. If data are grouped in an odd number of intervals, will the median always lie in the middle interval? Explain.
2. Give an example of grouped data for which the mode does not lie in the modal interval.
3. Make a new frequency table for the data in today's motivation problem, with fifteen tallies in each interval (0-14, 15-29, 30-44...). Which interval contains the mean? ...the median? ...the mode? Which is the modal interval? Does the modal interval contain the mean?

FREQUENCY TABLE #1

INTERVAL	FREQUENCY
40-49	6
50-59	1
60-79	15
80-82	8
83-89	17
90-94	7
95-99	6

FREQUENCY TABLE #2

INTERVAL	FREQUENCY
40-42	2
43-45	2
46-48	2
49-51	0
52-54	0
55-57	1
58-60	0
61-63	0
64-66	0
67-69	4
70-72	3
73-75	1
76-78	6
79-81	9
82-84	8
85-87	3
88-90	8
91-93	4
94-96	4
97-99	3

FREQUENCY TABLE #3

INTERVAL	FREQUENCY
40-59	6
50-59	1
60-69	4
70-79	11
80-89	25
90-99	13

FREQUENCY TABLE #4

INTERVAL	FREQUENCY
40-69	11
70-79	49

TOPIC 23: Algebra and Geometry

Aim: What is geometry?

Performance Objective: Students will be able to:

- . describe briefly the branch of mathematics called geometry.
- . classify angles (types of angles, parts of an angle).
- . classify triangles (types of triangles, parts of a triangle).

Vocabulary: point, line, plane, undefined term, line segment, ray, measure of an angle, vertex, right angle and triangle, acute angle and triangle, obtuse angle and triangle, scalene, isosceles, equilateral

Requirement:

The teacher should be completely familiar with the software before this lesson is taught. It would be more interesting for the student to see the material, beginning with this first lesson, in a new setting. Computer lab - 16 Apple IIe computers - 2 students at each.

Materials:

Chalkboard and chalk or special marker for new writing boards
Hardware - 16 Apple IIe
Software - "Introduction to Geometry" by SVE Software

Motivation:

Ask: "What is geometry?" Elicit many student opinions.

Development:

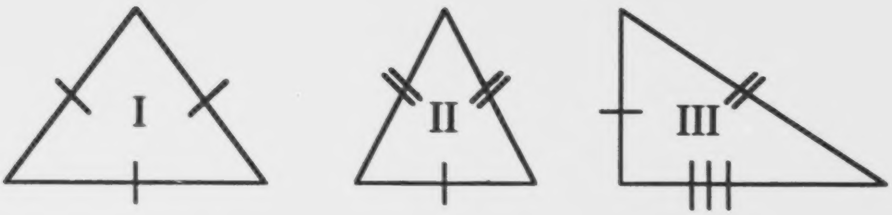
1. Using the software we will be in the geobotics industrial complex where geometric shapes are used to make robots.

2. The software allows the student to input his/her name. The student is then allowed to name a ray, line segment and line as well as different kinds of angles as they are moved into different kinds of positions. There is a help part of the software where the student may request help and review of concepts.

3. The student gets immediate feedback and points for correct answers. The idea is to create a robot using the geometric shapes identified and to see how many points are gained in the process.

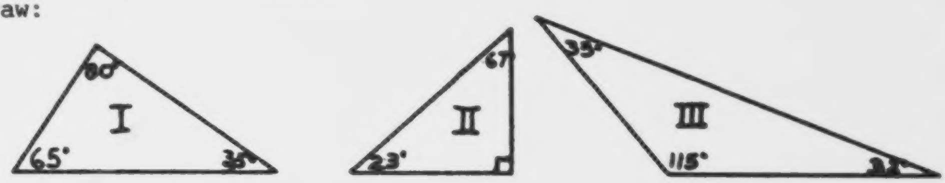
4. The piece of software allows a nice break in the lesson and tests the students' knowledge, which can be used in a medial summary.

a.



Look at the sides of these triangles. What seems to be true about △I? Elicit: The lengths of all three of its sides are equal. (Equi means "=". Lateral means "side.") State that this is an equilateral triangle and write the definition. Similarly, elicit that △II is isosceles and write its definition (that two sides are equal). Ask, "Is △ I also isosceles?" (Yes) Write its definition (that two sides are equal). Elicit that △ III has sides of three different lengths. It is called a scalene triangle.

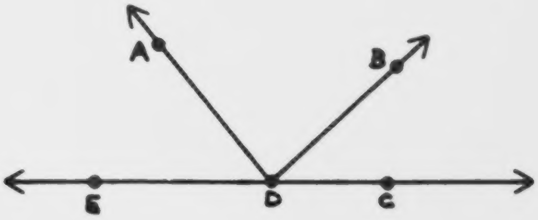
b. State: Triangles may also be classified according to the size of the angles they contain.
Draw:



Ask: What is notable about the angles in these triangles? Elicit: △ I has all acute angles, it is called an acute triangle. △ II has a right angle. What is it called? It is a right triangle. △III has an obtuse angle, therefore it is an obtuse triangle.

Applications

In this figure,



1. Name five points.

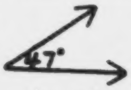
2. Name a line and five line segments.

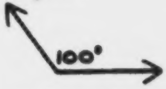
3. Name four rays.

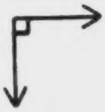
4. Name three angles containing $\overrightarrow{DA}$.

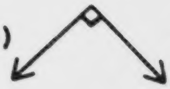
5. Name three angles that don't contain $\overrightarrow{DA}$.

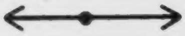
6. Name the kind of angle:

a)


b)


c)


d)


e)


7. Triangle ABC has angles that measure 50 degrees, 60 degrees and 70 degrees. What kind of triangle is △ ABC?

8. Name the type of triangles whose angle measures are listed below.
   a) 40 degrees, 50 degrees, 90 degrees b) 20 degrees, 50 degrees, 110 degrees c) 80 degrees, 80 degrees, 20 degrees

9. Name the types of triangles whose sides measure as follows.
   a) 12cm, 18cm, 20cm b) 30cm, 50cm, 30 cm c) 5cm, 5cm, 5cm

Summary:

1. Can a triangle be both isosceles and equilateral? Explain.

2. What are the types of angles we have learned so far?

3. Draw a triangle which is obtuse and scalene.

4. Draw a triangle which is obtuse and isosceles.

Homework:

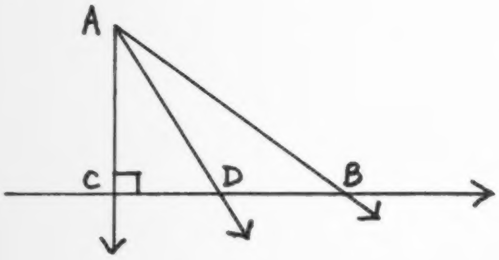
1. Your dictionary defines "point". Why do we classify "point" as undefined in geometry?

2. Show four points, A, B, C, D
   a) all in a line.
   b) at most, 3 in a line.
   c) at most, 2 in a line.
   d) How many lines are determined when at most 2 of the 4 points are in a line? (Draw them.)

3. Show 5 points, A, B, C, D, E, with at most 2 points in one line. How many line segments can you draw connecting 2 of these 5 points? Fill in the table as far as you can and try to discover a pattern.

Number of Points	1	2	3	4	5	6
Number of Line Segments						

4.



a. Name 3 triangles and classify them according to angles they contain.

b. Name one angle for each type learned so far.

c. Name all the line segments.

d. Name 6 rays.

e. Name one line.

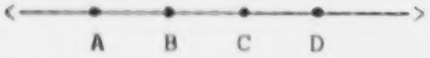
5. How many equilateral triangles can you make with 6 toothpicks, each a side of at least one triangle?

6. Can a triangle have more than one
   a) right angle?   b) obtuse angle?

7. Can an isosceles triangle also be a right triangle? Explain.

8. Can a right triangle be scalene? Equilateral? Explain.

9. How many line segments are in the figure:

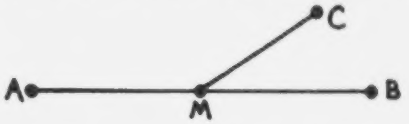


Fill in the table and try to find a pattern.

No. of points in the line	1	2	3	4	5	6
No. of line segments	0	1	3	6	10	15

10. Draw a triangle that is both
    a) obtuse and isosceles.
    b) obtuse and scalene.
    c) acute and scalene.

11. How many degrees are there in ∡AMC + ∡CMB? (AB is a straight line.)



12. What is the difference between a straight line and a straight angle?

13. In isosceles triangle ABC, $\overline{AB}$ and $\overline{BC}$ have equal lengths. If AB = 3x + 2, AC = 5x - 10, BC = 2x + 12, find AB, AC, BC.

TOPIC 24: Complementary and Supplementary Angles

Aim: What are complementary and supplementary angles?

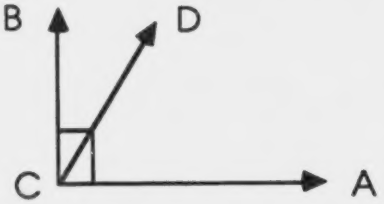
Performance Objectives: Students will be able to:

- define:
  - a) complementary angles
  - b) supplementary angles
- solve numerical and algebraic problems dealing with complementary angles, and supplementary angles.

Vocabulary: complementary angles, supplementary angles, adjacent angles

Motivation:

If $m\angle ACD = 70^\circ$, what is the measure of $\angle BCD$?



Materials:

Computer Program - The Geometric Presupposer, published by Sunburst Communications, for the Apple II family of computers.

Development:

1. Once the program is running on each computer, the students will be able to accomplish most of their tasks by using the menus which appear at the bottom of the screen.

   Students should indicate on their computers that they will be creating a NEW shape. Direct students to select the DRAW command and then to select ANGLE. The students should then indicate to the computer that they want it to draw a RIGHT angle. The angle will be labeled by the computer. In order to create two complementary angles, the students should do the following. From the menu on the bottom of the screen, select LABEL. Then select MOVABLE POINT. Using the arrow keys, the students should then move this new point to a location inside the existing right angle. They should then instruct the computer to DRAW a SEGMENT through the vertex of the angle and this newly defined point. The students may define the length of the segment, if so desired, by moving a flashing mark to each of the two endpoints, and pressing the RETURN key. At this point the teacher should elicit from the class the definition of the term adjacent angles: two angles with a common vertex and a common side between them and no interior points in common. Next, the students will direct the computers to MEASURE an ANGLE. When asked for the name of the angle, they will key in the name of one of the two adjacent angles.

2. Without using the computer, how will you find the number of degrees in the angle which is adjacent to the one which you have just measured? Students should state that there are 90 degrees in a right angle, and that if we subtract the measurement which was previously found from 90^c degrees, we will find the measure of the adjacent angle. Using paper and pen or pencil, students should perform this operation. Their result may be checked individually by having the computer measure this second angle. Students may also have the computer measure one of these angles, then press the "+" key and then key in the name of the second angle. The computer will indicate the individual measures as well as their sum.

3. If the sum of the measures of two angles equals 90°, what do we call these angles? COMPLEMENTARY angles.

4. Have students state the definition - two angles are called complementary angles if and only if the sum of their measures equals 90°.

5. If two adjacent angles form a right angle, may they be called complementary angles? Yes.

6. If the sum of the measures of two angles equals 90°, but the angles are not adjacent, are these angles complementary? Yes - the definition says nothing about the orientation of the angles, but only requires that the measures add up to 90°.

7. If the sum of the measures of three angles equals 90°, are the angles complementary? No - only two angles can be complementary.

8. Students may ERASE the line segment which divides the right angle and may then, following the procedures outlined above, create different pairs of complementary angles to measure. Teacher may determine how many times this should be done while he/she circulates to observe student work.

9. Using the ESC (if necessary) to return to the first menu, students should indicate that they are ready to create a NEW shape. They should then indicate that the computer should DRAW a SEGMENT. The computer will label two endpoints of the line segment. Following the procedures outlined for complementary angles the student should first LABEL a MOVABLE POINT which should be placed somewhere along the segment. (At this point, it would be appropriate to review the definition of a straight angle as having 180 degrees, and to identify the straight angle which has been created on each computer screen.) Another point should be placed either above or below the segment. Another segment should be drawn between these two new points. The students should then be instructed to have the computer measure one of the two resulting angles.

10. Now that you know the measure of one of these angles, without the computer, find the measure of the other angle. What is the sum of these two angles? (Students may be permitted to use the computer to check their results as was done previously.)

11. What do we call two angles whose measures total 180° ?
Supplementary angles.
Elicit: If two adjacent angles form a straight angle, then they are supplementary.

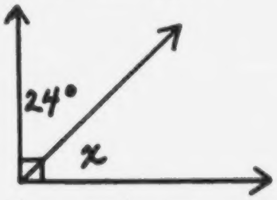
12. If the sum of the measures of three angles totals 180°, may these angles be called supplementary angles? No.

13. Students may ERASE the line segment which divides the straight angle and may then, following the procedures outlined above, create different pairs of supplementary angles to measure. Teacher may determine how many times this should be done while he/she circulates to observe student work.

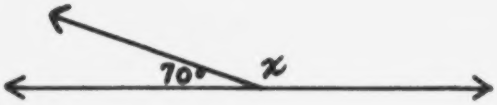
Application:

1. Find x in each case.

a)



b)



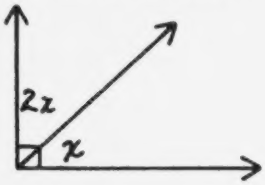
2. Find the complement and the supplement of each angle.
   a) 35 degrees   b) $19\frac{1}{2}$ degrees   c) x degrees

3. If one of two complementary angles measures 80°, what is the measure of the other?

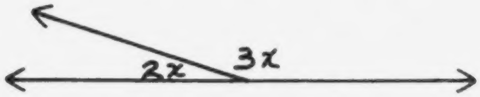
4. If one of a pair of supplementary angles measures 80°, what is the measure of the other?

5. Find x in each case, then find the measure of each angle.

a)



b)



6. Two complementary angles measure $(x + 15)^\circ$ and $(3x - 5)^\circ$. Find the measure for each angle.

7. Two supplementary angles measure $(2y - 12)^\circ$ and $(5y + 17)^\circ$. Find the measure of each angle.

8. One angle measures 20° more than its supplement. Find the measure of each angle.

9. If an angle measures 9 less than twice its complement, find the measure of the larger angle.

10. If the measure of two supplementary angles are in the ratio of 5:4, find the measure of the larger angle.

Summary:

1. What are complementary angles?

2. What are supplementary angles?

3. What angle has the same measure as its complement?

4. What is the supplement of the supplement of an angle whose measure is 15°?

5. Which is larger, the complement of 32 degrees or the supplement of 32 degrees? How much larger? Will this be true of all angles?

Homework:

1. The ratio of two complementary angles is 2:3. What are their measures?

2. One complementary angle is 10 degrees more than twice the other. What is the measure of the smaller angle?

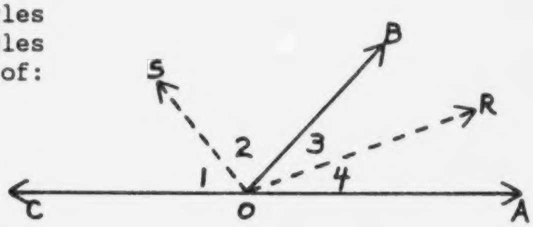
3. The ratio of two supplementary angles is 3:5. How much does each measure?

4. One supplementary angle is 30 degrees less than three times the other. How large is the greater angle?

5. $\overrightarrow{AOC}$ is a straight line $.\overrightarrow{OR}$ bisects ∢AOB (divides it into two equal parts). $\overrightarrow{OS}$ bisects ∢BOC

   a) Show that $\overrightarrow{OS} \perp \overrightarrow{OR}$.
   b) Name 3 pairs of complementary angles
   c) Name 3 pairs of supplementary angles
   d) If m∢AOB = 50, find the measures of:

      1) ∢AOR
      2) ∢BOC
      3) ∢BOS
      4) ∢AOS
      5) ∢ROS
      6) ∢ROC
      7) ∢BOR
      8) ∢COS



6. A quadrilateral is inscribed in a circle (all four of its vertices lie on the circle). Show that the opposite angles are supplementary.

7. If an angle is acute, what type of angle will its complement be? What type of angle will its supplement be?

8. What is the difference in the measures between the supplement and complement of an angle of 75° ?...of 25° ?...of n° ?

9. a) When is an angle equal to its own complement?
   b) When is an angle equal to twice its own complement?
   c) When is an angle equal to its own supplement?
   d) When is an angle equal to three times its supplement?

10. What is the sum of the measures of the supplement and complement of an angle of 75° ?...of 25° ?...of n° ?

11. The supplement of an angle is 4 times its complement. How large is the angle?

12. The supplement of the complement of an angle is 100°. How large is this angle?

13. The ratio of the complement of an angle to its supplement is 2:5. How large is this angle?

14. TRUE or FALSE. Explain your answer.

    a) The supplement of an angle is always greater than its complement.
    b) The 3 angles of a triangle add to 180 degrees so they must be supplementary.

TOPIC 25: Parallel Lines

Aim: What are parallel lines?
What are the postulates and theorems related to parallel lines?

Performance Objectives: Students will be able to:

- define transversal.
- define alternate interior angles, corresponding angles, interior angles on the same side of the transversal.
- define parallel lines.
- informally and formally state the postulates and theorems related to parallel lines.
- solve numerical and algebraic problems dealing with parallel lines.

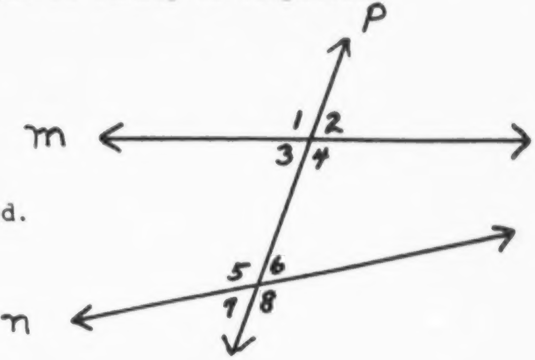
Vocabulary: transversal, alternate interior, corresponding angles, parallel lines.

Materials:

Computer Program - The Geometric Presupposer, published by Sunburst Communications, for the Apple II family of computers.

Motivation:

Name all pairs of angles in this figure which you could prove are congruent on the basis of what we have learned.



Development:

1. Students should identify four pairs of vertical angles which are congruent. (∡1 and ∡4, ∡2 and ∡3, ∡5 and ∡8, ∡6 and ∡7.)

2. Define: A transversal is a line that intersects two or more lines in distinct points.
   Ask, "Which line in this figure is a transversal?" (line p)

3. Ask: "How would you describe the positions of ∡4 and ∡5?" (nonadjacent, inside, and on opposite sides of the transversal) Ask, "What would you call these? (alternate interior angles) Are there any other pairs of alternate interior angles?" (∡3, ∡6)

4. Ask, "How would you describe the positions of ∡1 and ∡5?"
   (On the same side of the transversal and above the lines m and n,

nonadjacent, one is interior and the other is exterior.) These are called corresponding angles. Ask, "Are there other pairs of corresponding angles?" (∡3 and ∡7, ∡2 and ∡6, ∡4 and ∡8)

5. "What would you call ∡3 and ∡5, ∡4 and ∡6?" (Interior angles on the same side of the transversal.)

6. Ask, "Under what circumstances do you think corresponding angles 2 and 6 would be congruent?" Students should suggest that this will happen when lines "m" and "n" are parallel.

   Elicit the definition: Parallel lines are lines in the same plane that do not intersect.

7. Ask, "If the lines were parallel, do you believe ∡2 and ∡6 would be the only pair of corresponding angles which are congruent?"

   In order to answer this and other questions about parallel lines, we will enlist the assistance of the computer. After each machine has been loaded with the program, the students should notice the menu of commands at the bottom of the screen. This menu will change as the students proceed through the program. In order to return to a previous menu, press the ESC key.

8. The students should indicate that they are about to create a NEW shape. They should select DRAW followed by SEGMENT. Next select LABEL followed by MOVABLE POINT. Using the arrow keys, move the new point to a location apart from the original segment. Select DRAW followed by PARALLEL. Answer the computer's questions instructing it to draw a line through the new point, that is parallel to the original line, using the appropriate letter labels. The lengths of this new line segment may be defined by moving selected points to the ends and pressing the RETURN key. Following the previous procedure, LABEL a POINT located on each of these two lines. Instruct the computer to DRAW a SEGMENT through these two points. It may be necessary to LABEL additional points on this transversal so that the angles to be looked at may be properly identified.

9. By using the MEASURE function and measuring the appropriate angles, the students will gather the information necessary to answer the rest of the questions posed in this lesson.

10. If the teacher so desires, the students may ERASE the transversal and DRAW new ones in order to assure themselves that the postulates and theorems derived in relation to parallel lines are in fact true.

Summary:

1. How would you define parallel lines?

2. Define "transversal."

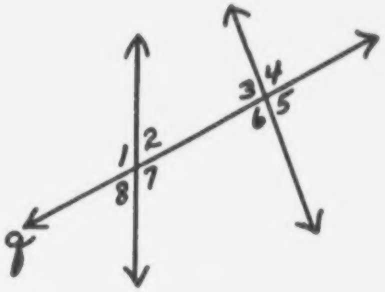
3. What are corresponding angles?

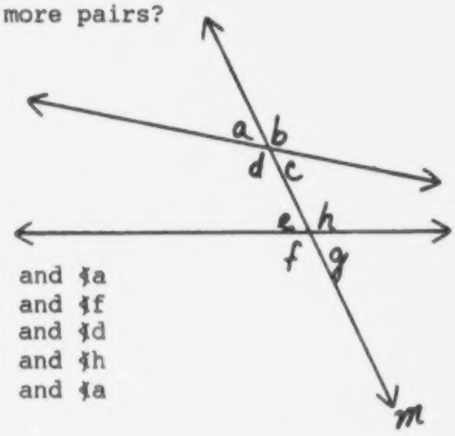
4. When two lines are cut by a transversal, how many pairs of alternate interior angles are formed?

5. Which postulate and theorems of parallel lines can we now state?

Homework:

1. For the figure at the right, name 2 pairs of angles that are: (q is the transversal)
   a) corresponding.
   b) alternate interior.
   c) alternate exterior.
   d) interior angles on the same side of the transversal.
   e) exterior angles on the same side of the transversal.
   f) equal in measure. Can you name two more pairs?
   g) supplementary.

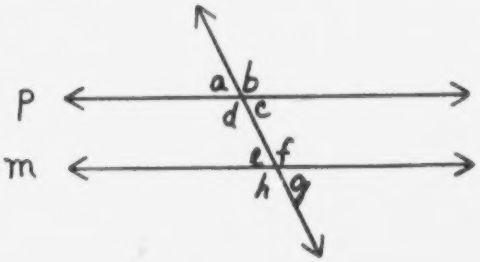




2. Describe the following pairs of angles: (The transversal is m.)
   a) ∢a and ∢e
   b) ∢a and ∢g
   c) ∢c and ∢e
   d) ∢c and ∢g
   e) ∢c and ∢h
   f) ∢c and ∢b
   g) ∢c and ∢a
   h) ∢b and ∢f
   i) ∢b and ∢d
   j) ∢b and ∢h
   k) ∢b and ∢a

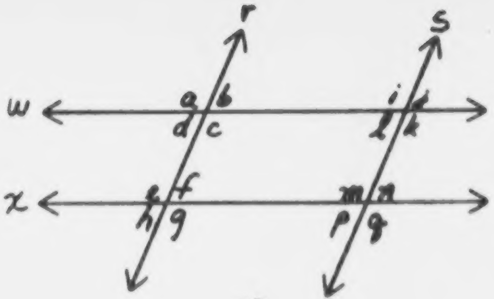
3. In the figure for ex. 2, suppose m∢a = 70° and m∢e = 80°. Find the measures of:
   a) ∢g
   b) ∢b
   c) ∢c
   d) ∢d
   e) ∢h
   f) ∢f

4. If $\overleftrightarrow{p} // \overleftrightarrow{m}$, what reason would you give for saying:
   a) m∢a = m∢e
   b) m∢c = m∢e
   c) m∢c = m∢g
   d) m∢b = m∢f
   e) m∢d = m∢f

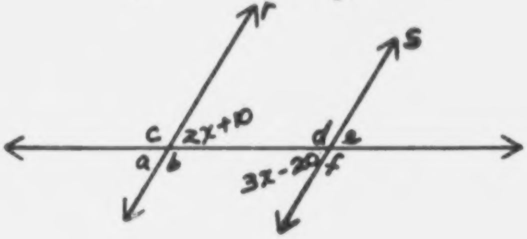


5. Using the information in ex. 4 and m∢a = 50°, find the measures of: ∢b, ∢c, ∢d, ∢e, ∢f, ∢g.

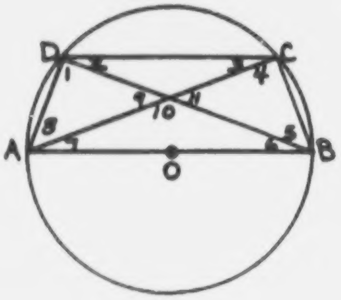
6. We are given that $\overleftrightarrow{w} \parallel \overleftrightarrow{x}$ and $\overleftrightarrow{r} \parallel \overleftrightarrow{s}$. Find all of the angles congruent to
   a) ∢a.
   b) ∢b.


r
s
w
a
b
d
c
i
j
l
k
x
e
f
h
g
m
n
p
q

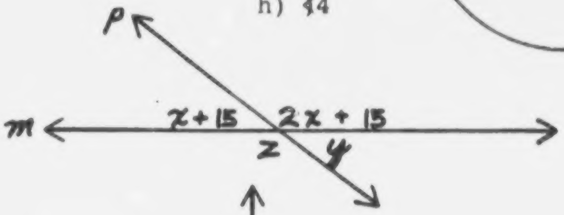
7. In the figure at the right with $\overleftrightarrow{r} \parallel \overleftrightarrow{s}$, find the degree measure of: ∢a, ∢b, ∢c, ∢d, ∢e, ∢f.


r
s
c
2x+10
a
b
d
e
3x-20
f

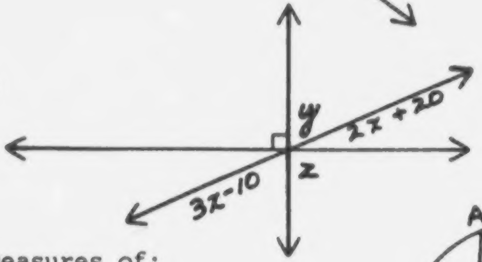
8. In the figure at the right diameter $\overline{AOB}$ is in a line parallel to $\overline{DC}$, and $m\overset{\frown}{AD} = 80°$. Find the measure of:
   a) ∢b
   b) ∢2
   c) $\overset{\frown}{CB}$
   d) $\overset{\frown}{DC}$
   e) ∢5
   f) ∢8
   g) ∢1
   h) ∢4


D
C
1
2
3
4
9
10
11
8
7
6
5
A
O
B

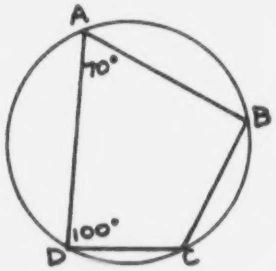
9. Solve for
   a) x
   b) y
   c) z


p
m
x + 15
2x + 15
z
y

10. Solve for:
   a) x
   b) y
   c) z


y
2x + 20
z
3x-10

11. Find the measures of:
   a) ∢C
   b) ∢B
   c) $\overset{\frown}{ABC}$
   d) $\overset{\frown}{BCD}$
   e) $\overset{\frown}{AB}$ if $\overset{\frown}{BC} = 80°$.



12. State the converses of the postulate and theorems discussed today. Do you believe the converses are true?

TOPIC 26: Sum of the Angles of a Triangle

Aim: What theorem can be stated regarding the measures of the angles of a triangle?

Performance Objectives: Students will be able to:

- state the sum of the angles theorem.
- informally and formally prove the theorem.
- solve numerical and algebraic problems involving the theorem.

Vocabulary: auxiliary line

Materials:

Apple computer lab
The Geometric Presupposer, published by Sunburst Communications, for the Apple II family of computers.

Motivation:

Teacher presents class with three dissimilar triangles (on chalkboard or on a duplicated page) and says: "Compare the sum of measures of the interior angles of each of these triangles with the sum of the measures of the interior angles of the other two."

Development:

1. After the program has been loaded into all of the computers in the classroom, the students should be instructed to notice the menu of commands at the bottom of the screen. Other appropriate menus will appear in this position throughout the program. To return to a previous menu, the student must press the ESC key.

2. Students are to indicate to the computer that they are going to create a NEW shape. They will then instruct the computer to DRAW, followed by the instruction TRIANGLE. On the next menu, the students should indicate that the triangle is to be a RIGHT triangle. Students can then use the MEASURE command to measure each of the angles of the triangle. They should then add these measures to determine the total number of degrees in the angles of the triangle.

NOTE: In the case of the right triangle, the students may MEASURE and add the two acute angles in one step, by indicating the name of the first angle to be measured, selecting the "+" key when asked by the computer, indicating the name of the other angle, and then pressing the RETURN key. This procedure may NOT be used for more than two angles.

Ask: "What was the sum of the measures of the angles of the right triangle on your monitor screen?"

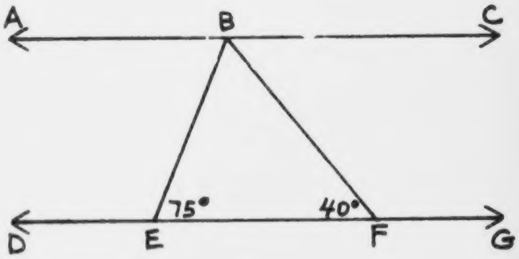
3. The students may then be asked to indicate a NEW shape and then to DRAW other triangles, subsequently measuring the angles and finding the sum of these measures.

4. After several triangles have been worked with, elicit the statement: The sum of the measures of the interior angles of a triangle equals 180°.

Optional:

5. How else might we prove this statement? At this point the teacher might use the computer program as follows:

Students DRAW a TRIANGLE. They then DRAW a SEGMENT through the base of the triangle, labeling the endpoints. They then DRAW a PARALLEL line through the opposite vertex, parallel to the base (in the diagram, through B, parallel to segment EF) after measuring the two base angles, the teacher should ask the students to find the measure of the third angle without using the computer. Once this has been accomplished and answers given, or if the students are having difficulty, the computer can be used to review the procedure step by step.



1. MEASURE each of the alternate interior angles.
2. Using previous knowledge about straight angles, determine how many degrees are in the third angle, which also happens to be the third angle of the triangle.
3. Check this result by having the computer MEASURE this angle.
4. Find the sum of the measures of the interior angles of the triangle.

Ask:

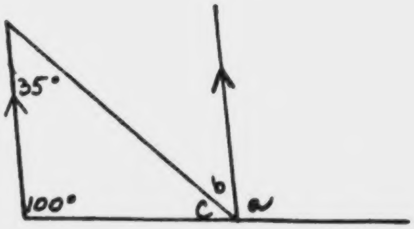
Now that we have proved the statement about the sum of the measures of the angles in a triangle, what are we entitled to call this statement? (a Theorem)

Summary:

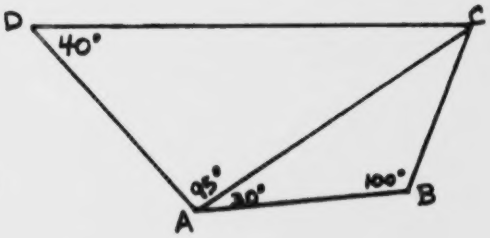
1. What is the theorem relating the measures of the angles of a triangle?
2. What earlier statements were needed to prove this theorem?

Homework:

1. Using the figure at the right, find the measure of:
   a) ∢a
   b) ∢b
   c) ∢c



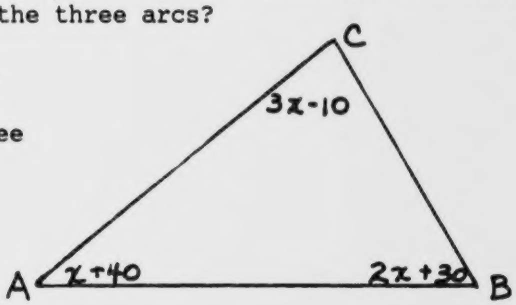
2. Using the figure at the right, find the measure of all 4 angles of the quadrilateral ABCD. What is the sum for all 4 angles?



3. The ratio of the measures of the angles of a triangle is 2:3:5. What are their measures?

4. If the triangle in example three is inscribed in a circle, what is the measure of each of the three arcs?

5. For the triangle at the right, find the measures of all three angles.



TOPIC 27: Quadrilaterals

Aim: What is a quadrilateral?

Performance Objectives: Students will be able to:

- define quadrilateral.
- classify quadrilaterals as parallelograms, rhombi (or rhombuses), rectangles, squares, trapezoids.
- define polygon.

Vocabulary: polygon, quadrilateral, parallelogram, rhombus, rectangle, square, trapezoid

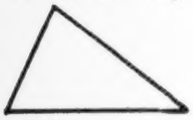
Materials:

Apple computer lab
Computer Program - Quadrilaterals, from the Scott, Foresman Geometry Courseware series, for the Apple II family of computers.

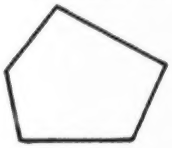
Motivation:

What do these figures have in common? How do they differ?

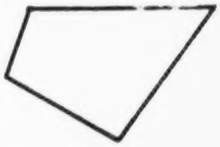
a)



b)



c)



Development:

1. What do all three figures have in common? Elicit: They are all closed figures made up of line segments. State: They are all polygons. Define: A polygon is a simple closed figure in a plane made up of line segments.

2. Elicit that polygons may be classified according to number of sides. Elicit that c) has four sides. Ask, "What is a quadrilateral?" Definition: A quadrilateral is a polygon with four sides.

3. Students will have had some experience with various special quadrilaterals.
   Ask, "Can you name any quadrilaterals which are special?"
   Ask students to draw the figures they name and to find examples in the room if possible.

   Load the Quadrilaterals program. After loading program, the students should choose Lesson 1, Types of Quadrilaterals. This program provides illustrated definitions of all of the quadrilaterals listed in the lesson. Students should be encouraged to record the definitions in their notebooks as they appear on the screen. Below are suggestions for making the best use of this program in this lesson.

4. After loading the appropriate lesson, students should press the RETURN key as indicated. When instructed to do so by the program, they should press L for a list of the definitions. The list is as follows:

1. Quadrilateral
2. Trapezoid
3. Parallelogram
4. Rhombus
5. Rectangle
6. Square

The students type the number of the shape which they would like to have defined. It is suggested that the students view the definitions in this order:

1. Quadrilateral
3. Parallelogram
5. Rectangle
4. Rhombus
6. Square
2. Trapezoid

5. The program mentions that a parallelogram having four right angles is a rectangle. The teacher should ask: May we say that a parallelogram which has one right angle is a rectangle? (Yes, because the parallel lines force the other angles to a be right angles also.) What is the relationship between a rhombus and a square? (A square is a rhombus with a right angle.)

6. At this point, the students may continue with the program, going on to problems which involve identifying various quadrilaterals. The teacher, circulating among the students, will be able to see the cumulative results on each monitor screen after each question is answered.

Summary:

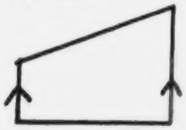
Consider the four sets: {Parallelograms}, {Rectangles}, {Rhombuses}, {Squares}.
1. Which set contains each of the other three?

2. Which set is contained in each of the other three?

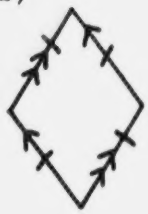
Homework:

1. Classify each of the following quadrilaterals.

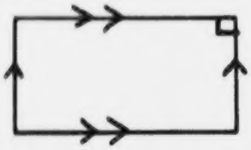
a)



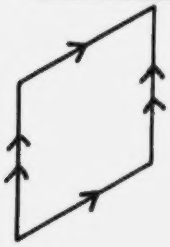
b)



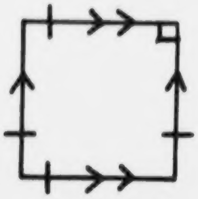
c)



d)

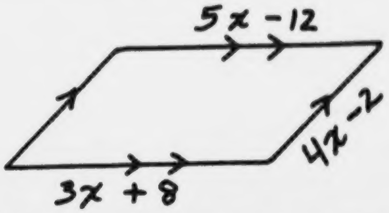


e)

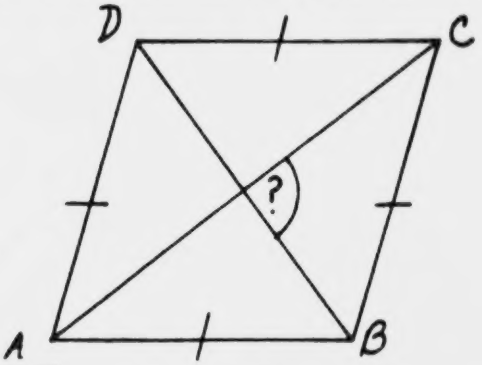


2. True or false. Try to explain.
   a) A trapezoid must have at least one acute and one obtuse angle.
   b) A parallelogram must have at least one acute and one obtuse angle.

3. Three sides of a parallelogram have lengths as shown. Find the lengths of all 3 sides and decide whether it is a rhombus. (Note: Opposite sides in a parallelogram are congruent.)



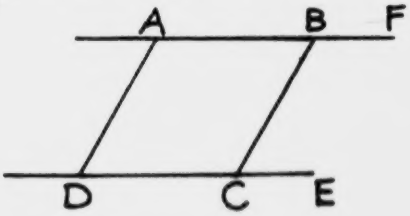
4. Draw carefully a rhombus ABCD and diagonals $\overline{AC}$ and $\overline{BD}$. Using your protractor, measure the angle made by the two diagonals. Conjecture a generalization.



5. Draw two line segments, $\overline{AB}$ and $\overline{CD}$, that bisect each other. Connect their endpoints to form a quadrilateral. Conjecture the type of quadrilateral we have.

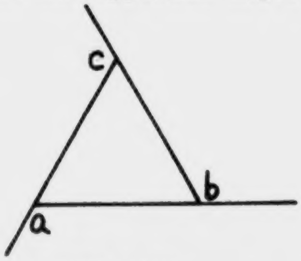
6. Draw two line segments, $\overline{AB}$ and $\overline{CD}$, that bisect each other (cut each other in half) and are perpendicular. Connect their endpoints. Conjecture the kind of quadrilateral we have.

7. $\boxed{P}$ ABCD has $m\angle BAD = 130^\circ$.
   Find the degree measure of:
   a) $\angle CBF$
   b) $\angle CBA$
   c) $\angle BCE$
   d) $\angle BCD$
   e) $\angle ADC$



8. What seems to be true about the measures of the opposite angles of a parallelogram? Try to prove this.

9. Draw any triangle and extend the sides as shown. Find $m\angle a + m\angle b. + m\angle c.$ What conjecture might you make about all such sums? Try to prove your conjecture.



10. Draw any quadrilateral. Join the midpoints of consecutive sides. What type of quadrilateral is formed?

11. Draw a rectangle. Join the midpoints of consecutive sides. What type of quadrilateral is formed?

12. Draw a rhombus. Join the midpoints of consecutive sides. What type of quadrilateral is formed?

If so desired by the teacher, additional homework questions may be found on the reproducible worksheet on page 16 of the Teachers' Guide for this program.

TOPIC 28: Measure of Angles of a Quadrilateral

Aim: What is the sum of the measures of the angles of a quadrilateral?

Performance Objectives: Students will be able to:

- recall that the sum of the measures of the interior angles of a triangle is 180 .
- state that the sum of the measures of the interior angles of a quadrilateral is 360 .
- solve numerical and algebraic problems involving the measures of the angles of a quadrilateral.

Vocabulary: quadrilateral

Materials:

Apple computer lab
Computer Program - The Geometric Presupposer, published by Sunburst Communications for the Apple II family of computers.

Motivation:

For each of the following, draw the figure and tell what the sum of its angles would be.
1. rectangle
2. parallelogram
3. trapezoid

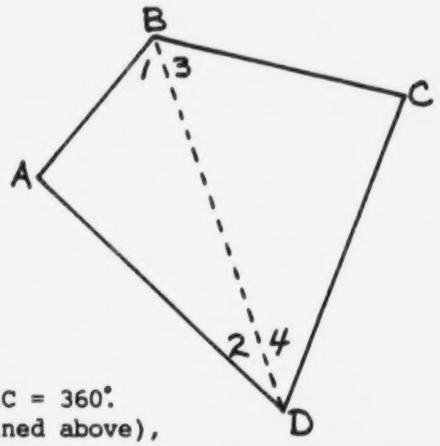
Development:

1. Discuss motivation. Students should observe that the sum in a rectangle is 4(90 degrees) or 360 degrees.
   The following argument applies for a parallelogram or a trapezoid: since same-side interior angles are supplementary, the sum is 2(180 degrees) or 360 degrees.

2. Ask: "Will the result be the same for every quadrilateral?" Explain. After each machine has been loaded with the program, the students should notice the menu to commands at the bottom of the screen. This menu will change as the students proceed through the program. In order to return to a previous menu, press the ESC key.

3. The students should indicate that they are about to create a NEW shape. They should then indicate that the shape will be a QUADRILATERAL. At this point, the teacher may have the students instruct the computer to draw a PARALLELOGRAM, a TRAPEZOID, or a RANDOM quadrilateral. The time saved by using this program should allow the students to experience each of these shapes. Once the shape has been drawn, the students instruct the computer to MEASURE each ANGLE, using the appropriate three letter name. When all four angles have been measured, the students should add the measures to see that their sum is 360 degrees.

4. Have the students indicate a NEW shape. Indicate QUADRILATERAL followed by the instruction RANDOM. In order to add the auxiliary line, instruct the computer to DRAW a SEGMENT from point B to point D. Then have the computer MEASURE the necessary ANGLES.



In $\triangle BAD$, $m\angle 1 + m\angle 2 + m\angle A = 180^\circ$.
In $\triangle BCD$, $m\angle 3 + m\angle 4 + m\angle C = 180^\circ$.
Adding equals to equals, we find
$m\angle 1 + M\angle 2 + m\angle 3 + m\angle 4 + m\angle A + M\angle C = 360^\circ$.
So (making replacements as explained above),
$m\angle A + m\angle B + M\angle C + M\angle D = 360$.
The statement has been proved, and may be regarded as a theorem.

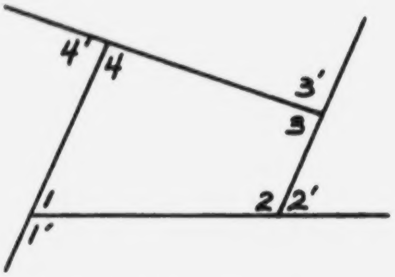
Summary:

1. How many straight angles would be needed to give the same sum as the measures in the four interior angles of a quadrilateral?

2. What is the sum of the measures of the interior angles of a pentagon?

Homework:

1. The ratio of the measures of the angles of a quadrilateral is 3:4:5:6. Find all four measures.



2. Suppose:
   $m\angle 1 = m\angle 2 - 10$
   $m\angle 3 = m\angle 2 + 10$
   $m\angle 4 = 2m\angle 1$
   Find the measures of angles 1,2,3 and 4.
   Find the sum of the measures of $\angle 1' + \angle 2' + \angle 3' + \angle 4'$.
   Conjecture a generalization about such sums for quadrilaterals. Try to prove your conjecture.

3. Show that if ▱ ABCD is inscribed in a circle, it must be a rectangle.

4. Draw a quadrilateral having
   a) 4 right angles
   b) 3 acute angles
   c) 3 obtuse angles
   d) 2 right angles and an obtuse angle.

5. a) Can a quadrilateral have 4 acute angles? Explain.
   b) Can a quadrilateral have 4 obtuse angles? Explain.

TOPIC 29: Properties of a Parallelogram

Aim: What are the properties of a parallelogram?

Performance Objectives: Students will be able to:

- state the properties of a parallelogram.
- solve numerical and algebraic problems involving the properties of a parallelogram.

Vocabulary: opposite sides, opposite angles, consecutive angles, diagonal, bisect

Materials:

Apple computer lab
Computer Program - The Geometric Presupposer, published by Sunburst Communications for the Apple II family of computers.

Motivation:

Today, the speed of the computer will assist our learning about parallelograms.

Development:

1. After each machine has been loaded with the program, the students should notice the menu of commands at the bottom of the screen. This menu will change as the students proceed through the program. In order to return to a previous menu, press the ESC key.
2. The students should indicate that they are about to create a NEW shape. They should then select QUADRILATERAL, followed by PARALLELOGRAM. They may then use the MEASURE command to measure ANGLES and SEGMENTS. The students will see that their findings apply to all parallelograms by creating several on the screen (one at a time) and taking the appropriate measurements.
3. If the teacher would like to have the students create their own parallelogram, they should follow the instructions below.
4. Indicate the creation of a NEW shape. Then select DRAW followed by SEGMENT. When prompted for the length of the segment, any length may be selected, but keep in mind that if a very small number is selected the segment may be difficult to work with. The segment will be labeled AB by the computer. Instruct the computer to LABEL a MOVABLE POINT. Use the arrow keys to move the point to a point on the screen either above or below point A. Press the RETURN key and the computer will name the new point C. Instruct the computer to DRAW a SEGMENT named AC. Next, select DRAW, followed by PARALLEL. Answer the computer's questions by instructing it to draw a line through point C, which is parallel to AB. There is no need to define the length of this new line segment. Instruct the computer to DRAW another PARALLEL line, this one through point B, parallel to AC. Define the length of this line by a MOVABLE POINT. Using the

arrow keys, move this point to where the two newly drawn lines meet. Press RETURN twice and the computer will assign a name to this point as well. The students may then carry out the appropriate measurements, knowing that they created the parallelogram and not the computer.

Summary:

1. What is true about the angles of a parallelogram?

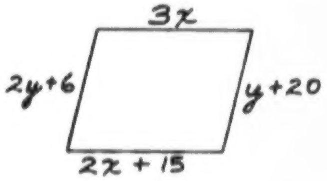
2. What is true about the sides of a parallelogram?

3. What is true about the diagonals of a parallelogram?

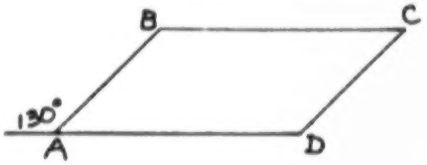
4. Why is it useful to be aware of these properties?

Homework:

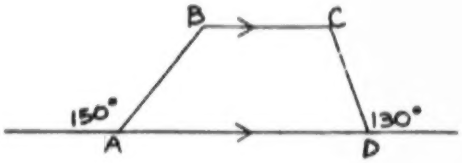
1. Find the perimeter of the parallelogram at the right. (A numerical answer is sought.)



2. Find the measures of all the angles of the parallelogram at the right.



3. Find the measure of all the angles of the trapezoid.



4. Name 4 quadrilaterals that must have or may have two opposite sides that are congruent.

5. If one angle of a parallelogram is 4 times some other angle, what are their measures?

TOPIC 30: Determine if a Quadrilateral is a Parallelogram

Aim: When is a quadrilateral a parallelogram?

Performance Objectives: Students will be able to:

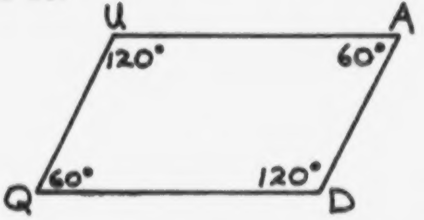
. state conditions sufficient for a quadrilateral to be a parallelogram.
. recognize a parallelogram from a marked diagram.

Materials:

Apple computer lab
Computer Program - The Geometric preSupposer, published by Sunburst Communications for the Apple II family of computers.

Motivation:

The measure of the angles of a quadrilateral, in order, are 2x, 4x, x + 30 and 5x - 30. What kind of quadrilateral is it?



Development:

1. Discuss motivation. Students should recall that the sum of the measures of the angles in any quadrilateral is 360 . This leads to the equation 12x = 360 . Therefore x = 30 degrees and the angles, in order, are 60 degrees, 120 degrees, 60 degrees and 120 degrees. Students should observe that the opposite angles are congruent. "What kind of quadrilateral is it?" Elicit that we have previously established that, in a parallelogram, the opposite angles are congruent. (If a quadrilateral is a parallelogram, then the opposite angles in the quadrilateral are congruent.) Is the converse of this statement true?

   After each machine has been loaded with the program, the students should notice the menu of commands at the bottom of the screen. This menu will change as the students proceed through the program. In order to return to a previous menu, press the ESC key.

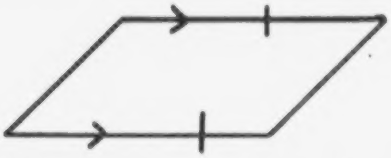
2. The students should indicate that they are about to create a NEW shape. They should then select QUADRILATERAL, followed by PARALLELOGRAM and RANDOM. This last selection insures that students do not all have identical parallelograms, allowing them to generalize from their findings. Students may observe this fact by looking at the monitor screens of their neighbors. The students should indicate the name of the segment to be measured followed by three strokes on the RETURN key. This last procedure should be repeated for each of the sides of the parallelogram. After measuring the fourth side, the RETURN key should be pressed only two times. Since the students know that the shape in question is a parallelogram, then they can be led to the conclusion: If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.

3. The students should next depress the ESC key. The computer is still prepared to do some measuring. Next, select ANGLE, indicate the name of

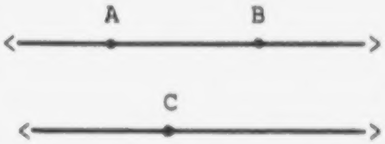
the angle to be measured, and press the RETURN key three times. This procedure should be followed for each of the angles of the parallelogram. After measuring the fourth angle, the RETURN key should be pressed only two times. The students may now be led to the conclusion: If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.

At this point, the students should press ESC twice and select END. When the program ends, the students should remove the disks and turn off the computers.

4. Ask, "Do you think that there are any other sets of conditions which would force a quadrilateral to be a parallelogram?" Consider the figure:



What do the markings indicate? (That one pair of opposite sides is both parallel and congruent.) Ask, "Do you think that this quadrilateral must be a parallelogram?"
State: Let's try to draw a figure that fits these conditions. Using your ruler and protractor, draw two parallel lines. Pick two points on one of the lines (A,B). Pick a point C on the second line and measure the distance between points A and B. Find point D to the right of C on the second line such that AB is congruent with CD. Now draw AC and BD. What kind of figure do we have? Students should use rulers and protractors to confirm that the figures they draw have all the properties of a parallelogram.



Elicit Method 3: If one pair of opposite sides of a quadrilateral is both parallel and congruent, the quadrilateral is a parallelogram.

Summary:

1. When is a quadrilateral a parallelogram? (State three sets of conditions, each of which being met would guarantee that the quadrilateral is a parallelogram.)

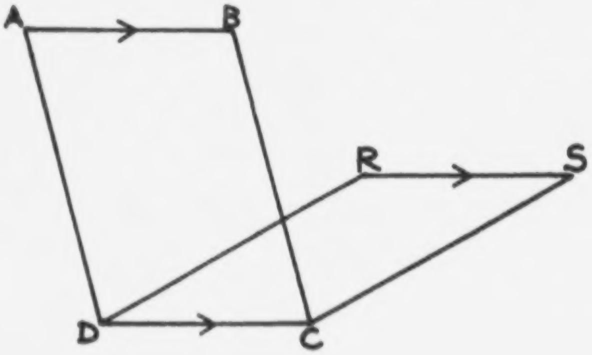
2. Why is it useful to know this? (If you recognize that a figure is a parallelogram, then you know that it has special properties. This information can be useful!)

Homework:

1. In quadrilateral ABCD, $\overleftrightarrow{AB}$ // $\overleftrightarrow{DC}$, AB = DC. Regarding this figure, state:
   a) The kind of quadrilateral ABCD is.
   b) A relation between consecutive angles.
   c) Three equalities regarding its sides and angles.
   d) A relation between $\overline{AD}$ and $\overline{BC}$ not previously mentioned.

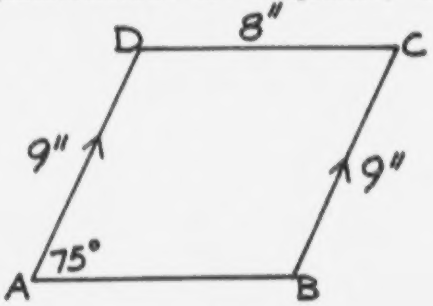
2. In the figure described above, if AD = 7", what must be the length of $\overline{BC}$? Explain.

3. In the figure at the right $\overline{AB}$ // $\overline{DC}$, $\overline{RS}$ // $\overline{DC}$, AB = DC, RS = DC.
   What relation holds between:
   a) $\overline{AB}$ and $\overline{RS}$. Explain.
   b) AB and RS. Explain.
   c) $\overline{AD}$ and $\overline{BC}$. Explain.
   d) AD and BC. Explain.
   e) $\overline{RD}$ and $\overline{SC}$. Explain.
   f) RD and SC. Explain.

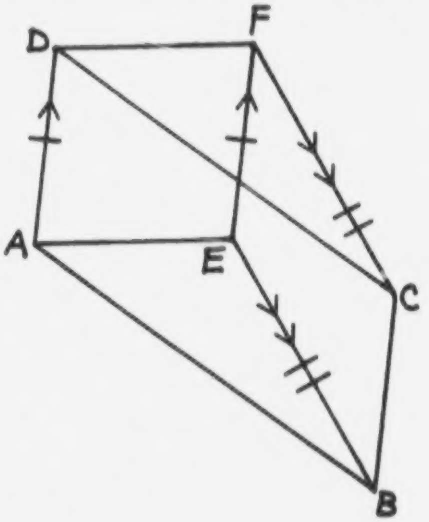


4. Answer TRUE or FALSE and explain:
   a) If $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are in the same plane.
   b) If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect, they must be parallel.
   c) If a quadrilateral has 2 parallel sides and 2 congruent sides, then it must be a ▱.
   d) If 3 angles of a quadrilateral are right angles, the fourth angle must also be a right angle.
   e) If all 4 angles of a quadrilateral are congruent, then it must be a rectangle.

5. Find the measure of the following:
   a) $\measuredangle B$
   b) $\measuredangle D$
   c) $\measuredangle C$
   d) AB



6. In the figure shown at the right, explain why
   a) AEFD is a ▱
   b) EFCB is a ▱
   c) $\overline{EF}$ // $\overline{BC}$
   d) EF = $\overline{BC}$
   e) $\overline{EF}$ // $\overline{AD}$
   f) EF = AD
   g) AD = BC
   h) $\overline{AD}$ // $\overline{BC}$
   i) DC = AB
   j) $\overline{DC}$ // $\overline{AB}$



TOPIC 31: Properties of Other Quadrilaterals

Aim: What are the properties of a rectangle, square and rhombus?

Performance Objective: Students will be able to:

- list the properties of a rectangle, square and rhombus.
- solve numerical and algebraic problems involving the properties of these quadrilaterals.

Materials:

Apple computer lab
Computer Program - Quadrilaterals, from the Scott Foresman Geometry Courseware series, for the Apple II family of computers.
The students will use Lesson 5 on the disk for this session.

Motivation:

What is the relationship between parallelograms and rectangles?

Development:

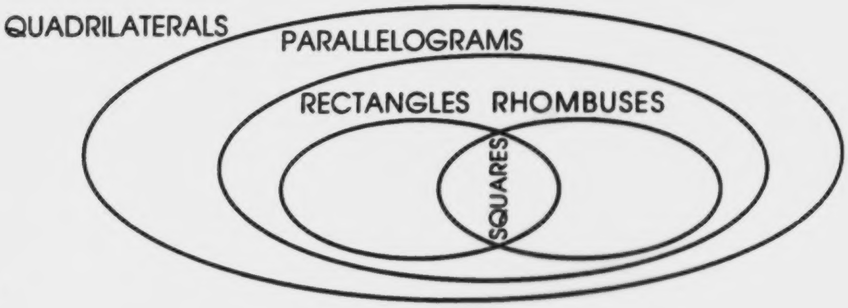
1. The definitions in this program are clear and the illustrations are very helpful in enabling the student to visualize the information being presented.

2. The students should work through the definitions and samples as the teacher circulates and assists. The program works very well with students working in pairs. Students should be guided in taking notes, and definitions/properties should be outlined on the board.

   When the students have completed the program, the disks should be removed and the computers turned off.

3. Ask: "What is the relationship of a square to the quadrilaterals (parallelogram, rectangle, rhombus) observed? Elicit that a square is all of them; it is a parallelogram, a rectangle and a rhombus. Therefore, a square has all the properties previously discussed. Elicit and list all the properties of a square.
   1) The sum of the measures of its interior angles is 360 degrees.
   2) Opposite sides are parallel.
   3) Opposite sides are congruent.
   4) Opposite angles are congruent.
   5) Diagonals bisect each other.
   6) All angles are right angles.
   7) Diagonals are congruent.
   8) All sides are congruent.
   9) Diagonals are perpendicular.
   10) Diagonals bisect the angles of the quadrilateral.
   11) Consecutive angles are supplementary.

4. Complete the diagram begun earlier, showing all special quadrilaterals discussed in this lesson.



Summary:

1. What is the most special quadrilateral? (a square)

2. Examine the list of properties of a square. Which of its properties are due to the fact that it is a quadrilateral? (#1) Which of its properties are due to the fact that it is a parallelogram? (#2,3,4,5,11) ...a rectangle? (#6,7) ...a rhombus? (#8,9,10)

Homework:

1. If $m\angle A = 113$ in ▱ ABCD, find the measures of the other three angles.

2. In rhombus ABCD, the length of side $\overline{AB}$ = the length of diagonal $\overline{AC}$ = 9". Find the measures of:
   a) The other sides of the rhombus.
   b) The 4 angles of the rhombus.

3. In rectangle ABCD, $AB = 3x + 2$, $BC = 2y + 7$, $CD = 6x - 10$, $DA = 4y$. What is the measure of each side? What is another way to name this quadrilateral?

4. Every rectangle is a parallelogram. Is this statement true? State the converse of this statement. Is the converse true? (If yes, explain why. If not, give an example which proves the statement is false.)

5. If a quadrilateral is not in a plane, we say that it is a skew quadrilateral. Admitting skew quadrilaterals, is it possible for such a quadrilateral to have
   a) each of its four angles 30 degrees? Using toothpicks, show such a quadrilateral if possible.
   b) only three of its four angles 90 degrees and the fourth an acute angle? Show such a quadrilateral with toothpicks if possible.
   c) same as b but the fourth angle obtuse?

Additional or alternate homework questions may be found on page 20 of the computer program users guide.

TOPIC 32: Trapezoids

Aim: What are the properties of a trapezoid?
What is an isosceles trapezoid?

Performance Objectives: Students will be able to:

- list the properties of a trapezoid.
- define an isosceles trapezoid.
- list the properties of an isosceles trapezoid.
- solve numerical and algebraic problems involving the properties of trapezoids.

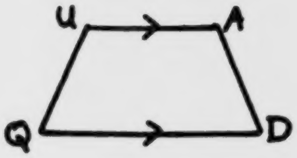
Vocabulary: trapezoid, bases, legs, isosceles trapezoid

Materials:

Apple computer lab
Computer Program - The Geometric Presupposer, published by Sunburst Communications, for the Apple II family of computers.

Motivation:

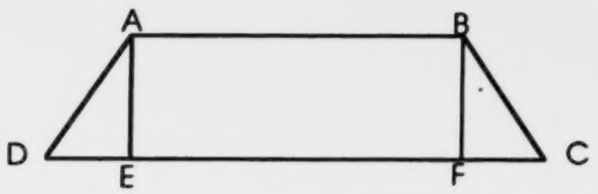
Is QUAD a trapezoid?



Development:

1. After each machine has been loaded with the program, the students should notice the menu of commands at the bottom of the screen. This menu will change as the students proceed through the program. In order to return to a previous menu, press the ESC key.

2. The students should indicate that they are about to create a NEW shape. They should then select QUADRILATERAL, followed by TRAPEZOID and RANDOM. If the trapezoid which has been drawn is perceived as too small by the students, they may select SCALE in order to enlarge it. If they then find the trapezoid to be too big for the screen, they may again select SCALE to return the trapezoid to its original size. Once this procedure has been completed the students should instruct the computer to DRAW a PERPENDICULAR line. The line should be perpendicular to the base of the trapezoid, while going through a vertex on the opposite side. This procedure should be repeated, following the clear menu choices so that two altitudes have been drawn between the two sides which the students perceive to be parallel.



3. After completing the diagram, so that it resembles the illustration above, the students should indicate that they want to MEASURE the LENGTH of each altitude, by entering the names of the appropriate segments, one at a time. Upon discovering that the altitudes are of equal length, the students should realize that the two sides in question are in fact parallel, because parallel lines are everywhere equidistant. By simply looking at the trapezoid, the students can tell that the other two sides are not parallel. Therefore, a trapezoid is a quadrilateral with one and only one pair of parallel sides. Furthermore, the parallel sides of a trapezoid are called its bases and the nonparallel sides are called legs.

4. What will be true about the two angles on the top and bottom of either leg of a trapezoid (∢A and ∢D; ∢B and ∢C in our illustration)? (They are supplementary since interior angles on the same side of a transversal that intersects two parallel lines are supplementary.) In order to demonstrate this last statement, the students should indicate that they want to MEASURE and ANGLE, giving the appropriate three letter name for each angle in question. After an angle is measured, press RETURN three times to measure another angle. When all measuring is completed, press RETURN twice.

5. Is it possible for a trapezoid to have a pair of opposite sides which are congruent? Students should return to the first menu, using the ESC key, and should then indicate that they want to create a NEW shape. Subsequently they should indicate QUADRILATERAL, TRAPEZOID, and ISOSCELES. Next, MEASURE the LENGTH of each base and of each leg to determine that the answer to the above question is yes, but only if they are the nonparallel sides. (If the parallel sides were also congruent, it would force the figure to be a parallelogram.) Elicit: a trapezoid with two congruent legs is called an isosceles trapezoid.

6. Using the isosceles trapezoid on the computer screen, the students should next MEASURE each ANGLE using the procedure outlined previously. Elicit the fact that the base angles of an isosceles trapezoid are congruent. (Note: Each trapezoid has two pairs of base angles.)

7. Again using the existing isosceles trapezoid, the students should DRAW a SEGMENT connecting the opposite corners of the trapezoid (a diagonal) by naming the two points as the endpoints of the segment to be drawn. They should repeat this procedure to draw the other diagonal. Next have the computer MEASURE the LENGTH of each segment. Elicit: The diagonals of an isosceles trapezoid are congruent.

Summary:

1. How does a trapezoid differ from a parallelogram?

2. What is an isosceles trapezoid?

3. What are the special properties of an isosceles trapezoid?

4. In the diagram of the set of quadrilaterals made in the previous lesson, where would you place the set of trapezoids? ...the set of isosceles trapezoids?

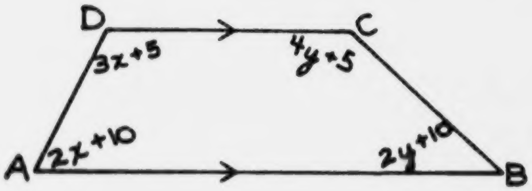
Homework:

1. One base angle of an isosceles trapezoid measures 63°. Find the measure of each of the other 3 angles?

2. For the trapezoid at the right, find:

   a) $m\angle A$ c) $m\angle D$

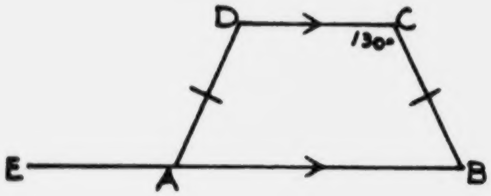
   b) $m\angle D$ d) $m\angle C$



3. For the trapezoid at the right, find the the following:

   a) $m\angle B$ c) $m\angle D$

   b) $m\angle A$ d) $m\angle EAD$

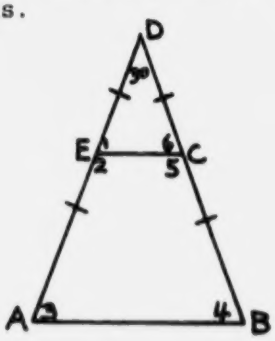


4. Is it possible for a trapezoid not to be completely in a plane? Explain.

5. If the two legs of a trapezoid are congruent, then we learned that:
   a) The base angles are congruent. State a converse. Do you think this converse is true?
   b) The diagonals are congruent. State a converse. Do you think this converse is true?

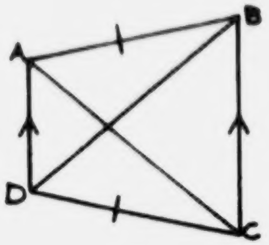
6. If the opposite sides of a quadrilateral are congruent, must it be a ▱P? Explain or show with toothpicks.

7. Find the measures of all six angles: 1,2,3,4,5,6. What seems to be true about $\overline{EC}$ and $\overline{AB}$? What kind of figure is ABCE?



8. For the figure at the right we have

   AC = 7a + 23
   BD = 5a + 111
   Find AC and BD.



9. Draw two line segments that are perpendicular to and bisect each other. Connect the four endpoints to form a quadrilateral. What kind of quadrilateral does it seem to be? Check by measuring.

10. If all four sides of a quadrilateral are congruent, must it be a rhombus? Explain or show by using toothpicks.

TOPIC 33: Congruent Polygons

Aim: What are congruent polygons?

Performance Objective: Students will be able to:

- define congruent polygons.
- recognize pairs of congruent triangles by SAS, ASA and SSS.

Vocabulary: congruent polygons

Materials:

Chalkboard, chalk
Hardware: Large Monitor attached to an Apple IIe computer.
Software: Intro to Geometry - SVE software

Motivation:

Use of the software in which congruence of many different shapes and congruent figures in different positions does not change congruence. Ask the question, "What is a clone?" The figures formed using the software form figures which are clones or congruent. Figures are then seen to have the same size and same shape, and this is the primary concept to be stressed here.

Development:

1. The teacher controls how fast the lesson will progress. The parts used which are congruent form cloned robots.

2. You get a second try at each pair - to determine whether or not the shapes are congruent.

3. The ultimate outcome of the questions is a pair of clone robots (another pair of congruent shapes).

4. It is worth the time to use this visual device to illustrate many different congruent shapes.

5. Students will probably ask for the measure of each angle and the length of each side. Offer three paper strips or sticks which are congruent to the sides of the given triangle. The students will see that they can be assembled in only one way. Explain that we have accomplished the objective using only 3 of the 6 parts of the triangle.

6. Define congruent ($\cong$) polygons as polygons having all pairs of "corresponding" sides congruent, and all pairs of "corresponding" angles congruent. Elicit that congruent polygons are also similar ($\sim$), but $\sim$ polygons are not always $\cong$. More generally, two $\sim$ polygons are congruent if they can be named ABC and A'B'C' so that $\angle A \cong \angle A'$, $\angle B \cong \angle B'$... and $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$... When are similar polygons congruent? (When the ratio of similitude is 1:1.)

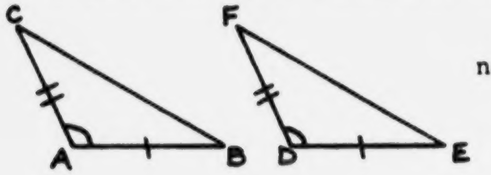
7. **Ask: "Will any three pairs of congruent parts suffice to guarantee congruent triangles?" Suppose only three angles were known. (Show a smaller or larger cutout triangle which is similar but not congruent to the original one.)**

8. **Divide the class into two groups. Ask one group to draw a line segment 11" long, a 60 degree angle at the end of the segment, and mark off 5 $\frac{1}{2}$" along the second ray of the angle. Make a triangle by connecting the ends of the segments. Have the other group draw the 11" segment with a 30 degree angle at one end and a 60 degree angle at the other. Extend the rays of each angle until they intersect.**

9. **Compare students' triangles. Cut several out from each group and show by measuring and/or "overlapping" that the triangles are ≅.**
   Ask, "Which combination of measures produced triangles which were congruent to the given triangle?" (3)
   List: Two triangles are congruent if
   a) Two sides and the included angle of one are ≅ to corresponding parts of the other (SAS).
   b) Two angles and the included side of one are ≅ to corresponding parts of the other (ASA).
   c) Three sides of one triangle are ≅ to corresponding sides of the other (SSS).

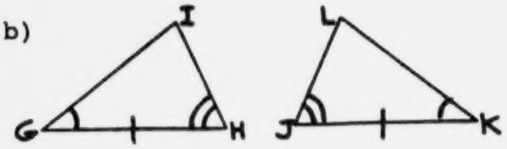
Therefore, given the facts marked off in each diagram, we can say:

a)



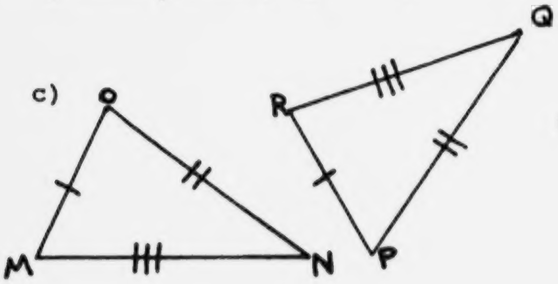
ABC ≅ DEF (SAS)
name corresponding parts

b)



GHI ≅ KJL (ASA)
name corresponding parts

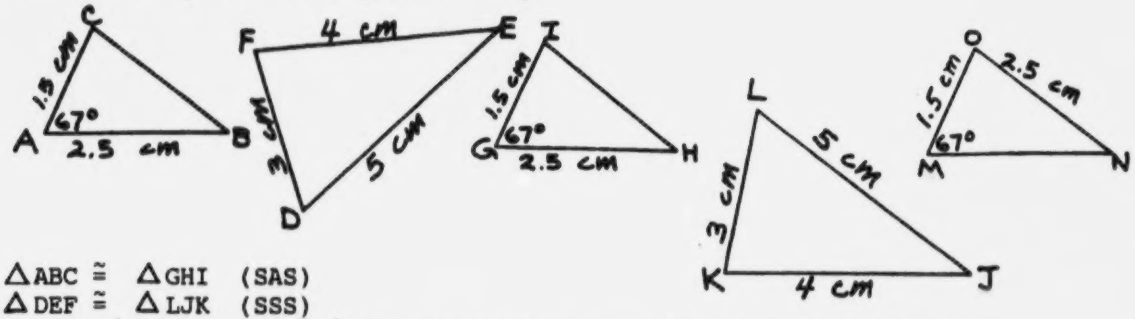
c)



MNO ≅ RQP (SSS)
name corresponding parts

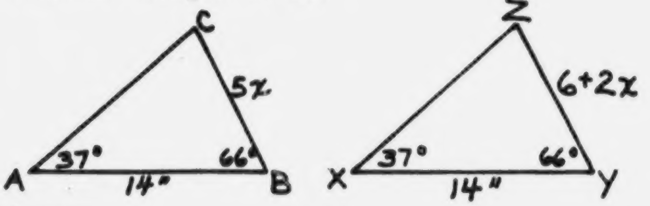
Applications:

1. Which triangles are congruent? Why?



$\triangle$ABC $\cong$ $\triangle$GHI (SAS)
$\triangle$DEF $\cong$ $\triangle$LJK (SSS)
Note: $\triangle$MNO is not $\cong$ $\triangle$ABC since 67 degrees is not included between the congruent sides.

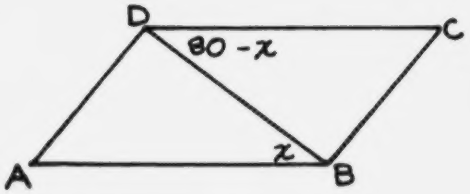
2. Is $\triangle$ ABC $\cong$ $\triangle$ XYZ? Why? Find the length of $\overline{BC}$.
(≅ by ASA, so
$5x = 6 + 2x$
$3x = 6,\ x = 2,\ 5x = 10$
BC = 10)



3. a) If ABCD is a parallelogram is $\triangle$ABD $\cong$ $\triangle$CDB?

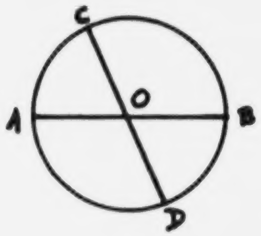
(Elicit $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$ and $\overline{BD} \cong \overline{DB}$, $\triangle \cong \triangle$ SSS)

b) Find ∢ ABD. ($x = 80 - x$, $2x = 80$, $x = 40°$ m∢ ABD = 40°)



Elicit: The diagonal of any parallelogram divides it into two congruent triangles.

4. If O is the center of the circle with diameters $\overline{AB}$ and $\overline{CD}$, what triangles can be shown to be ≅?
(Since all radii of circle O must be ≅ and since vertical angles are ≅, $\triangle$AOC $\cong$ $\triangle$BOD and $\triangle$AOD $\cong$ $\triangle$COB, both by SAS.)



Summary:

1. How can we establish that two triangles are congruent?

2. Why is it useful to be able to prove that triangles are congruent? (Since only three congruent pairs need be shown to prove that figures are congruent, another three pairs of congruent parts are a happy consequence.

3. How many/which pairs of parts would you have to show congruent to prove that two quadrilaterals are congruent?

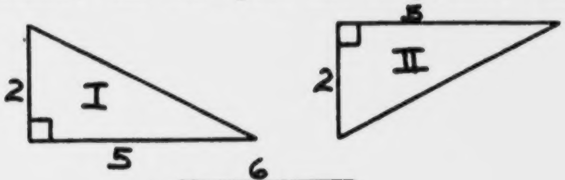
Homework:

1. a) According to the definition of congruence, what six equalities must hold for
$\triangle ABC \cong \triangle A'B'C'$?

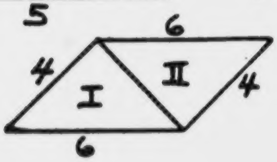
   b) What three equalities are sufficient for $\triangle ABC \cong \triangle A'B'C$? (We have learned of three sets of three equalities. Give all three.)

2. In each case below, the triangles are congruent. Explain why.

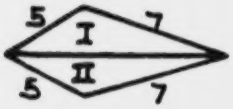
a)



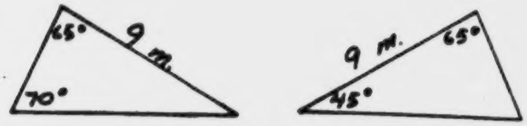
b)



c)

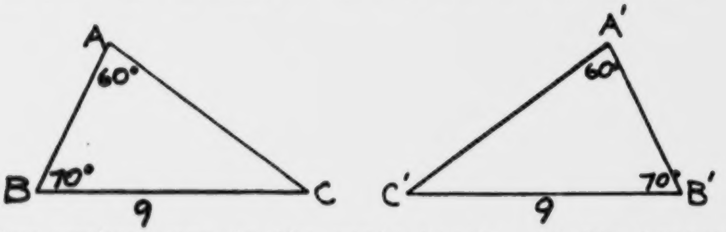


d)



3. If quadrilaterals XYZW and X'Y'Z'W'are congruent, what eight equalities must hold according to the definition of congruence?

4. How might you justify that $\triangle ABC$ and $\triangle A'B'C'$ are congruent?



5. Three positive numbers are in the ratio 3:4:5. If the largest number is 8 more than the smallest, what are the three numbers?

6. Suppose that for $\triangle ABC$ and $\triangle A'B'C'$ we have the following three equalities: $\measuredangle A = \measuredangle A'$, $\measuredangle B = \measuredangle B'$ and BC = B'C'. Must they be congruent? If not, show by example.

7. The diagonal of any parallelogram divides it into two congruent triangles.
   a) Write this conditional statement in if-then form.
   b) State its converse.
   c) Is the converse a true statement? Prove or show a counterexample.

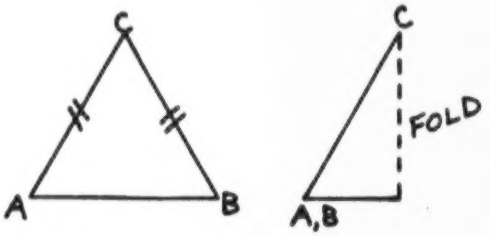
TOPIC 34: Properties of Isosceles Triangles

Aim: What are the properties of an isosceles triangle?

Performance Objectives: Students will be able to:

- state the base angles theorem.
- state the converse of the base angles theorem.
- solve numeric and algebraic problems involving isosceles triangles.

Motivation:



Ask each student to construct a triangle ABC with $\overline{AC} \cong \overline{BC}$. Then ask students to fold the paper so that point B coincides with point A. What can the class conclude about angle A and angle B?

Materials:

Apple computer lab
Computer Program - The Geometric Presupposer, published by Sunburst Communications, for the Apple II family of computers.

Development:

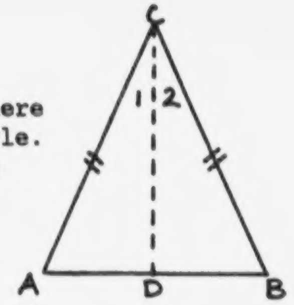
1. Elicit the statement that the base angles of an isosceles triangle are congruent. Review the concept of "theorem".
   Theorem: If two sides of a triangle are congruent, then the angles opposite them are congruent. Inform the students that they will now use the computer to test the theorem and to assist them in visualizing the concepts presented in this lesson.

2. After each machine has been loaded with the program, the students should notice the menu of commands at the bottom of the screen. This menu will change as the students proceed through the program. In order to return to a previous menu, press the ESC key.

3. The students should indicate that they are about to create a NEW shape. They should then select TRIANGLE followed by ISOSCELES. At this point they may use the MEASURE command to measure each of the sides of the triangle as well as each of its angles. They now have all of the needed information to see that the theorem presented and its converse are true.

4. Have the students select DRAW from the menu. Next, they select ANGLE BISECT, and when prompted to do so, give the name of the angle to be bisected, using the appropriate three letter name. The line segment which will bisect the angle will be DEFINED BY A MOVABLE POINT. At this

point, use the arrow keys to move this point to where the seqment intersects with the base of the triangle. Press the RETURN key twice. The students now have a diagram similar to the one at right. If the teacher desires, the students may now use the MEASURE command to measure each of the sides and each of the angles of the triangle.



Summary:

1. Ask class: "What conclusions can we reach concerning △ABC if we know $\overline{CA} \cong \overline{CB}$?" (Isosceles △, ∡A ≅ ∡B)

2. In XYZ what can we conclude if we know ∡X ≅ ∡Y? (Isosceles △, $\overline{ZX} \cong \overline{ZY}$)

3. If the vertex angle of an isosceles triangle is congruent to the vertex angle of another isosceles triangle, will the triangles be similar? (yes) Explain.

4. If a base angle of an isosceles triangle is congruent to the base angle of another isosceles triangle, will the triangles be similar? (yes) Explain.

5. If one angle of an isosceles triangle is congruent to one angle of another isosceles triangle, will the triangles be similar? (Not necessarily: the angle might be a vertex angle in one triangle and a base angle in the other.)

Homework:

1. The vertex angle of an isosceles triangle measures 40°. What is the measure of each base angle?

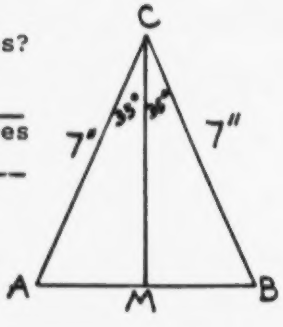
2. A base angle of an isosceles triangle measures 25°. What is the measure of the vertex angle?

3. Show that if all three angles of a triangle have the same measure, all three sides must have the same measure.

4. Which of the following triangles are isosceles?

	∡A	∡B	∡C
a)	45 degrees	90 degrees	
b)	60 degrees		60 degrees
c)	56 degrees	63 degrees	

5. Show that $\overline{AM} \cong \overline{BM}$ in the figure at the right.



TOPIC 35: Area of a Rectangle or a Triangle

Aim: How do we find the area of a rectangle or triangle?

Performance Objectives: Students will be able to:

- define area.
- find the area of a:
  - a) rectangle
  - b) triangle

Materials:

Computer lab: Apple or MS DOS machines
Computer Program - Geometry Alive, disk entitled Areas of Triangles & Quadrilaterals, published by Educational Activities, for Apple or MS DOS computers.

Development:

This disk is divided into three sections. This lesson will use section B, Rectangles & Triangles.

Note: Section A, The Concept of Area, is a good introduction to (or review of) the concept of area and is recommended for use by an entire class or by individuals needing such an introduction or a review. The teacher should review this section before reaching such a decision.

1. Before beginning to use the program, review the concept of area and of square units of measure. Ask: "What does area mean?" An informal definition might be the amount of room in a closed figure (in a plane or two dimensions). Just as the length of a line segment is measured in units (inch, cm, etc.) we have specific units to measure area. (Using your notebook as a unit of area you could find the area of your desktop.) Common units of measure are square inch, square centimeter, square yard etc. A square inch is the area of a square whose side is one inch long.

2. The program may then be loaded into the computers. The students should select B, as was previously mentioned.

   The program initially addresses the area of a rectangle. It develops the formula, A = bh, using clear illustrations and explanations. Each step used in finding the area of a rectangle is clearly illustrated:

   1. Write the formula
   2. Substitute the appropriate values
   3. Multiply

3. The program then moves on to triangles. It begins with a right triangle, which is represented as being equal to one half of a rectangle. The area of a right triangle is one half of the area of a rectangle with the same base and height. "That ought to suggest a formula" - gives the student time for thought. It then presents the formula $A = \frac{1}{2}bh$. The steps used in finding the area of a triangle are illustrated:

    1. Write the formula
    2. Substitute the appropriate values
    3. Find one-half of one of the factors
    4. Multiply

4. In order to find the area of an acute triangle, the program introduces the concept of the altitude. It then follows a procedure similar to that used for right triangles.

5. When addressing the question of the area of an obtuse triangle, the program presents the concept of the altitude falling outside of the triangle. Although the program once again relates the area of a triangle to the area of a rectangle, the method used here is slightly different, so that the students must concentrate in order not to be confused.

   Practice examples are provided at the end of each section.

Summary:

1. What is the area of a basketball court 50' by 84'? (4200 sq. ft.)

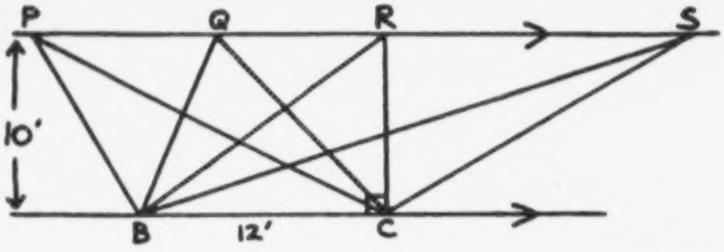
2. How do we find the area of a triangle?

Homework:

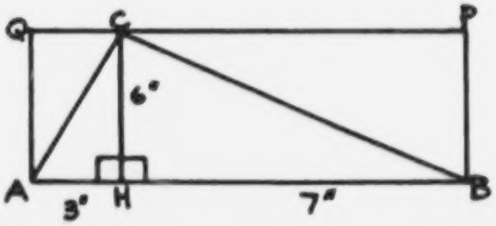
1. A rectangle has a perimeter of P in. and an area of A sq.in. It is enlarged by a factor of 5, so its length and width are each multiplied by 5. How are its perimeter and area changed?

2. The length of a rectangle is three less than twice its width. If the length is increased by 3 and the width is decreased by 1, a new rectangle is formed whose area is 5 more than the original rectangle. Find:
   a) the dimensions of the original rectangle.
   b) the area of the new rectangle.

3. What is the area of each triangle?
   a) △PBC
   b) △QBC
   c) △RBC
   d) △SBC



4. By finding the area of the smaller right triangles, △AHC AND△CHB, show that the area of△ABC is one-half the area of the rectangle ABPQ.



5. Show (without using numbers) that when the altitude falls inside the triangle, as in example three above, that the area of the triangle will always be one-half the area of the rectangle.

TOPIC 36: Area of a Parallelogram or a Trapezoid

Aim: How do we find the area of parallelograms and trapezoids?

Performance Objectives: Students will be able to:

- find the area of a parallelogram.
- find the area of a trapezoid.

Materials:

Apple or MS DOS computer lab
Computer Program - Geometry Alive, disk entitled Areas of Triangles & Quadrilaterals, published by Educational Activities, for Apple or MS DOS computers.

Development:

This program is divided into three sections. This lesson will use section C, Squares, Parallelograms, Trapezoids.

1. The first shape presented in this program is the square. The square is presented as a special rectangle, having four equal sides. The formula for the area of a rectangle is reviewed and is converted into the formula the area of a square, $A = s^2$. This new formula is then explained, as is the concept of s^2 being equal to (s)(s). The program then illustrates the steps for finding the area of a square:

    1. Write the formula
    2. Show its meaning [(s)(s)]
    3. Substitute for s
    4. Multiply

2. The program next moves on to the area of a parallelogram. After drawing a parallelogram, the computer draws an altitude to represent the height of the parallelogram. It then moves the newly created triangle to the other side of the parallelogram, thereby creating a rectangle with the same area as the original parallelogram. The program goes on to develop the fact that therefore the area of a parallelogram may be represented by $A=bh$ as may the area of a rectangle. The program then illustrates the steps for finding the area of a parallelogram:

    1. Write the formula
    2. Substitute the appropriate values
    3. Multiply

3. The section of the program which presents the area of a trapezoid, is not presented in a fashion similar to the way in which most teachers would present it to their classes. It is strongly suggested that the teacher preview this section and then make the decision as to whether or not to use it with their classes.

4. The program begins by drawing a trapezoid with an altitude to represent the height. It names the top base b_1 and the bottom base b_2. The program then draws an altitude on the other side of the trapezoid. This allows the subsequent separation of the two resulting triangles from the trapezoid, leaving two right triangles and a rectangle. The two right triangles are joined to create one new triangle. We are told that the base of the new triangle is equal to $b_2 - b_1$. We are then told that the area of the trapezoid is equal to the area of the rectangle added to the area of the triangle, as follows:

   Area of rectangle = bh   Area of triangle = $\frac{1}{2}$ bh

   Substitute $A = b_1h$ $A = \frac{1}{2} h (b_2 - b_1)$

   Add formulas   Area of a trapezoid = $b_1 h + \frac{1}{2} h(b_2 - b_1)$

   Distribute   Area of a trapezoid = $b_1h + \frac{1}{2} b_2h - \frac{1}{2} b_1h$

   Combine like terms   Area of a trapezoid = $\frac{1}{2} b_1 h + \frac{1}{2} b_2h$

   Factor   Area of a trapezoid = $\frac{1}{2} h(b_1 + b_2)$

   The steps used in finding the area of a trapezoid are then illustrated:

   1. Write the formula
   2. Substitute the appropriate values
   3. Find one half of the height
   4. Add the length of the two bases
   5. Multiply

   Mixed practice, covering each of the area formulas learned is available on the disk if the teacher wishes to use it.

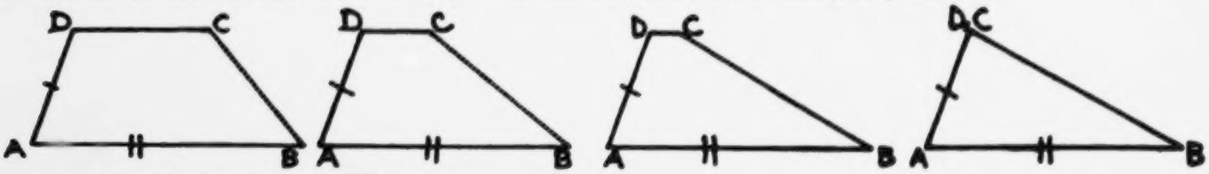
Summary:

1. How can we find the area of a parallelogram? (base x height)

2. What is meant by the height of a parallelogram?

3. How do we find the area of a trapezoid?

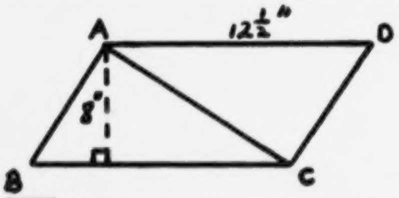
4. In trapezoid ABCD, what happens if CD shrinks to a single point?"


D
C
A
B
D
C
A
B
D
C
A
B
DC
A
B

What will the length of $\overline{CD}$ become? (0)
What kind of figure will we have? (△ABD)
In the area formula for a trapezoid, what is the result if a base equals zero? ($\frac{1}{2}h(b\ )$ or $\frac{1}{2}bh$, the area formula for a triangle.)

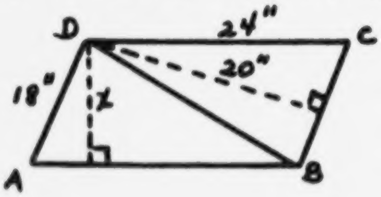
Homework:

1. Find the area of △ABC if ABCD is a parallelogram.


A
$12\frac{1}{2}$"
D
8"
B
C

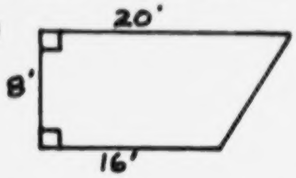
2. If the area of ▱ ABCD is 288 sq. ft. and AB = 15 ft., what is its height as measured to $\overline{AB}$?

3. Find the height of ▱ ABCD to $\overline{AB}$ using the measurements shown in the figure.

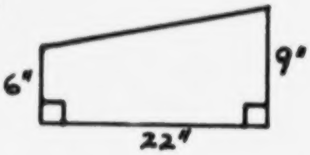

D
24"
C
18"
x
20"
A
B

4. Find the area of each trapezoid.

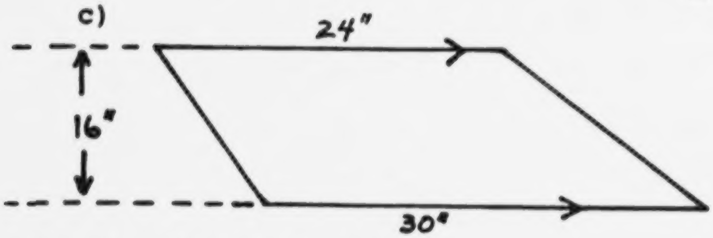
a)



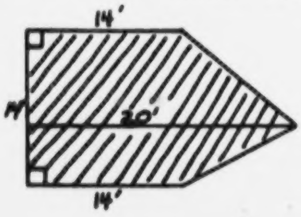
b)

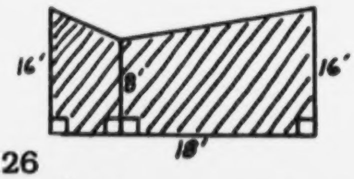


c)



5. Find the area of each shaded region.





TOPIC 37: Rational Numbers

Aim: What is a Rational Number?

Performance Objectives: Students will be able to:

- define a rational number.
- list the types of numbers that can be written as a fraction (whole numbers, fractions, terminating decimals, nonterminating repeating decimals).

Vocabulary: rational number, terminating decimal, nonterminating repeating decimal, real number

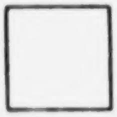
Materials:

Chalkboard, chalk
Hardware: 1 large monitor hooked up to an Apple IIe
Software: Practical Algebra Manual - Intellectual software.
Choose code L30

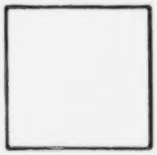
Motivation:

For each square below, find the length of one side.

Area = 4 sq. in.



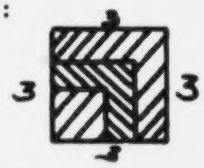
Area = 6 sq. in.



Area = 9 sq. in.

Development:

1. Discuss motivation. We are looking for the positive root of $x^2 = 4$, $x^2 = 6$ and $x^2 = 9$. The first square has a 2" side; the third square has a 3" side. The center square must have a side whose length is between 2" and 3" since the squares could be nested like this: Elicit that we may try to "trap" the side of the square between two numbers which are less than one inch apart. Ask, "Will the side of the middle square be longer or shorter than 2"?" Since $(2\frac{1}{2})^2 = (2\frac{1}{2})(2\frac{1}{2})$ or $(2.5)(2.5) = 6.25$ or $6\frac{1}{4}$, it is clear that the side will be shorter than $2\frac{1}{2}$. Try several other approximations suggested by the students. Ask, "If we continue with this method, do you think we will find the exact answer? Does an exact answer exist?" (Yes, the square is real.) The length of the side of the middle square may be designated $\sqrt{6}$ (read: the square root of 6). This number is approximately equal to 2.4494897



2. Have the students in the classroom view the above software (code L30 is an option on the menu).

3. Have the students define rational number. Then view the definition. This makes the writing on the board unnecessary.

4. After viewing the definition review parts (use of Cloze).

   a. Rational number is a fraction, integer, or mixed number.
   b. The top of the fraction is the numerator and the bottom is the denominator.
   c. An integer may always be expressed as having a denominator of 1.

Applications:

1. These exercises test for rational number and integer, just a rational number, an imaginary number, or none of these.

2. The software goes on to illustrate how a rational number is changed to a decimal.

3. Terminating decimals end.

4. Other rational numbers lead to a repeating, nonterminating pattern of digits. An illustration is given.

5. The exercises ask the student to find the repeating patterns. A management system is built in. It is User Friendly in that it asks the student's name. A second try is allowed when a wrong answer is given.

6. An assortment of other applications are given which are very good. Have students input responses into computer and other students work at the chalkboard.

Summary:

1. How is the set of whole numbers related to the set of rational numbers?

2. How do we define a rational number in terms of integers?

3. Why is an integer considered to be a rational number?

Homework:

1. State which of the given numbers is greater.

   $\frac{1}{3}$ or -2 $\qquad$ $\frac{5}{8}$ or $\frac{-12}{8}$ $\qquad$ $\frac{-10}{4}$ or $\frac{-13}{4}$

2. Name a rational number which lies between the two given numbers

   $\frac{1}{3}$ and $\frac{7}{9}$ $\qquad$ -2 and -3 $\qquad$ $-2\frac{1}{3}$ and -3

3. Write each rational number as a repeating decimal:

   $\frac{11}{5}$ $\qquad$ $\frac{7}{9}$ $\qquad$ $\frac{2}{9}$ $\qquad$ $\frac{1}{11}$

4. Tell whether the number is rational, irrational or whether there is insufficient information to tell.

5 $.4\overline{5}$ 1.265
9 .454454445... 1.265438...
.45 .4041 1.266555...

5. Express as a quotient of two integers:

a) 4.3

b) $.08\,\frac{1}{4}$

c) $.2\,\frac{1}{3}$

d) -.07

e) $.1\overline{3}$

f) $.3\overline{9}$

g) $\overline{.4}$

h) $.0\overline{4}$

i) 1.4

j) $1.\overline{4}$

k) $1.0\overline{4}$

l) $1.0\overline{4}$

TOPIC 38: Irrational Numbers

Aim: What are Irrational Numbers?

Performance Objective: Students will be able to:

- define irrational number.
- give examples of rational and irrational numbers.

Vocabulary: rational number, irrational number, terminating decimal, nonterminating nonrepeating decimal, principal square root, real number

Materials:

Chalkboard, chalk
Hardware: Large monitor hooked up to an Apple IIe computer
Software: Practical Algebra Manual - Intellectual software - use code L35 loaded and ready to go.

Development:

1. The concept that an irrational number cannot be written as a terminating decimal is highlighted.

2. The term perfect square is illustrated and the term perfect square root is stressed as follows:

   a. Even though $5 \times 5 = 25$, $-5 \times -5 = 25$ yet $\sqrt{25} = 5$ this is the principal square root.

   b. A set of exercises on the monitor helps to reinforce this concept.

3. Several square roots are given that can not be changed into an exact whole number to illustrate irrational numbers:

   a. $\sqrt{16}, \sqrt{27}, \sqrt{36}, \sqrt{45}$ nonrepeating decimals are seen. Also Pi (π), an irrational number is illustrated.

4. The introduction of principal square root here is appropriate.

   a. Irrational numbers are real numbers. Students should be able to visualize irrational numbers as points on the number line. e.g. $\sqrt{3}$

```
<--------¦------------¦------¦---¦--¦------>
         0            1     1.5 √3  2
```

5. Irrational numbers may be approximated by rational numbers. For example, $x = .38541\ldots \approx .39$.

   Define real numbers: The union of sets containing all rational numbers and all irrational numbers is the set of real numbers.

Application:

Tell whether the number is rational, irrational or whether there is not sufficient information to tell.

$.52$
$.525252...$
$.\overline{52}$
$.525525552...$
$.24389216$
$.124124124...$
$.52552555\overline{2}$
$-\overline{.621}$
π
$.7\overline{4}$
3
$.13121110987...$
6.25
$6 + \pi$

Summary:

1. Give an example of an irrational number.

2. State whether the statement is true or false:
    a) Every rational number corresponds to a point on the number line.
    b) There are some numbers that are both rational and irrational.
    c) Every irrational number corresponds to a point on the real number line.
    d) Every real number is rational.
    e) Every irrational number is real.

Homework:

1. State whether each of the following is rational, irrational or lacks sufficient information to tell:
    a) 2.3
    b) $2.\overline{3}$
    c) 2π
    d) $2 + \pi$
    e) $.1010010001...$ (The number of zeros increases by 1 in each group.)
    f) $.12321...$
    g) -17

2. Answer TRUE or FALSE or NOT TAUGHT YET.
    a) Every real number has a location point on the number line.
    b) Every point on the number line names some real number.
    c) 2 is an irrational number.
    d) The sum of two rational numbers is always a rational number.
    e) The sum of two irrational numbers is always an irrational number.
    f) The product of two rational numbers is always a rational number.
    g) The product of two irrational numbers is always an irrational number.
    h) The sum of a rational number and an irrational number is always an irrational number.

TOPIC 39: The Square Root of a Number

Aim: How do we find the square root of a number?

Performance Objective: Students will be able to:

- . estimate the square root of positive integers less than 100 to the nearest tenth.
- . compare positive and negative square roots of perfect squares.

Vocabulary: index, radical, radical sign, radicand, principal square root, perfect squares

Materials:

Chalkboard, chalk
Hardware: Large monitor attached to an Apple IIe
Software: Plato software: Elementary Alg-Polynomials, Control Data Pub.
Choice F on the menu.
Pract. Alg. Man-L40, Intellectual software

Motivation:

1. The area of a square is 25 sq. in. What is the length of a side?
2. What is the square root of 25?

Development:

1. Discuss motivation
   a. Elicit $\sqrt{25}$ = Side = 5
   b. The area of a square is 8 sq. units. What is the length of a side? $\sqrt{8}$ = side. At this point we would use the large monitor to see what happens - choose code L40.
2. A set of problems for which we would guess approximately the non-perfect squares is given.
   a. $\sqrt{20}$ 1) 4, 2) 4.5, 3) 5, 4) 5.5
   b. $\sqrt{34}$ 1) 5, 2) 5.5, 3) 6, 4) 6.5
3. Then the approximate square root method is given, namely, take the two closest whole numbers, add, find the average and divide the answer into the number for which you are trying to find the square root (pencil and paper needed here).
4. A formal method is then presented.

Applications:

1. Find the principal square root for each of the following: 1, 25, 81, 169, $\frac{1}{9}$, $\frac{9}{16}$, $\frac{49}{25}$, .81.

2. Express the radical as an integer, a fraction or a decimal.

   $\sqrt{4}$, $\sqrt{36}$, $-\sqrt{100}$, $\sqrt{0}$ $\sqrt{400}$, $\sqrt{.49}$

   $-\sqrt{625}$, $\sqrt{\frac{1}{4}}$, $\pm\sqrt{\frac{9}{16}}$, $\sqrt{\frac{25}{81}}$, $\pm\sqrt{144}$

3. Approximate the square roots to the nearest tenth.
   $\sqrt{80}$, $\sqrt{14}$, $\sqrt{3}$, $\sqrt{21}$

4. Find the length of the side of the square whose area is 9 sq. units.

5. Find to the nearest tenth, the side of the square whose area is 8.

6. Do the applications on the Plato software if time allows.

Summary:

1. How would you proceed to find an approximate value for $\sqrt{20}$?

2. Find the value of $(\sqrt{17})^2$. Explain.

3. Which is larger, $\sqrt{5}$ or $-\sqrt{5}$? Explain. Locate both numbers on the number line.

Homework:

1. Find the principal square root of: 9, 49, 121, 225.

2. Express the radical as an integer, fraction or decimal.

   $\sqrt{16}$ $\sqrt{64}$ $-\sqrt{144}$ $\pm\sqrt{196}$ $\sqrt{1}$

   $\sqrt{\frac{9}{4}}$ $\sqrt{.81}$ $\pm\sqrt{\frac{49}{100}}$ $\sqrt{36}$ $-\sqrt{81}$

3. Approximate the square root to the nearest tenth:

   $\sqrt{2}$ $\sqrt{24}$ $\sqrt{90}$ $\sqrt{61}$

4. Find to the nearest tenth the length of a side of a square whose area is 17.

5. Find the principal square root of each of the following... to the nearest tenth if not a perfect square.

a)	256	d)	.49	g)	$.\overline{4}$
b)	2.56	e)	.4	h)	.16
c)	.81	f)	.09	i)	.1

6. Use a divide and average method to obtain the square root of each of the following to the nearest hundredth. Be sure you have a sound argument for the correctness of your answer, excluding a calculator or table.

a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{5}$ d) $\sqrt{.9}$

7. State in words the meaning of $\sqrt{n}$.

8. From the meaning of $\sqrt{n}$, what is the value of

a) $\sqrt{3} \times \sqrt{3}$ d) $\sqrt{6.25} \times \sqrt{6.25}$

b) $\sqrt{.7} \times \sqrt{.7}$ e) $\sqrt{.1} \times \sqrt{.1}$

c) $\sqrt{625} \times \sqrt{625}$ f) $\sqrt{a} \times \sqrt{a}$

9. Find the perimeter of a square if its area is

a) 900 sq. in.
b) 169 sq. yd.
c) 841 sq. mi.

10. Which is a better approximation to $\sqrt{6.4}$, 2 or 3? To verify your prediction, find $\sqrt{6.4}$ to the nearest tenth.

TOPIC 40: Multiplication of Radicals

Aim: How do we multiply radicals?

Performance Objectives: Students will be able to:

- simplify monomial radicals
- multiply monomial radicals

Vocabulary: radicand, coefficient, simplify, factors

Material:

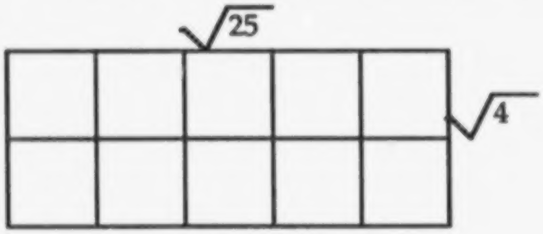
Apple IIe Program Elementary Algebra - Exponents , Roots & Radicals by Control Data Publishing, Lesson 3 Pt 2 Objective 3b

Motivation:

What is another way of writing $\sqrt{4}$? Answer: 2. What's another way of writing $\sqrt{25}$? Answer: 5. If we multiplied $\sqrt{25} \times \sqrt{4}$ What do you think the answer might be?

or Alternate Approach for Motivation

Could I express the sides of the given rectangle as follows:



Development:

1. For the problem the student will now turn to the relevant program on the computer. From the program itself students will view an illustration of the multiplication process. Ask what answer did the computer give us for the product of $\sqrt{25} \times \sqrt{4}$? How can we prove this? Give time for the student to view this illustration, then review once more the steps in proving $\sqrt{25} \times \sqrt{4} = \sqrt{100}$. Do this by asking: "What was the square root of 25... 4? What was the product of these two square roots?" If students seem to be having any difficulty have them view the next illustration on the computer by again following the same procedure as above (Allow rest to use D&F of this program)

2. Continue with this same line of reasoning by asking if the product of $\sqrt{25}$ and $\sqrt{4}$ is $\sqrt{100}$, would it follow that 2 terms which are not perfect squares could also have their products generated in the same way? In other words, can we take the approximate roots $\sqrt{8}$ and $\sqrt{2}$ multiply them together and create the approximate value of their product? Let's see... We should know by now that the approximate value of $\sqrt{8}$ is between 2 and 3. What can we say about the square root of two... its value? Using square root table and/or calculators, what is the value of $\sqrt{8}$? ... $\sqrt{2}$?

3. Let's multiply (2.828)(1.414)... Where do these numbers come from? What is their product? Let's multiply radical($\sqrt{8}$) and radical($\sqrt{2}$). What was the square root of these two radicals? Does this come close to the product using the approximate tables.

Generalization:

How did the computer show us the way to handle problems like $\sqrt{9} \times \sqrt{4}$?
What should we do first?... second?
Question: Let's make a general rule regarding $\sqrt{ab}$. What does our experience on the computer and the board tell us to do with something like this? Answer: $\sqrt{a} \times \sqrt{b}$.
Question: If we square $\sqrt{ab}$ what will we produce? How can we write $(\sqrt{ab})^2$ another way? Answer: $\sqrt{ab} \times \sqrt{ab}$
Question: Can we write this another way? How?
Answer: $\sqrt{a} \times \sqrt{a} \times \sqrt{b} \times \sqrt{b}$.
Question: What principle of mathematics allows this to happen?
Answer: The rearrangement principle.
Now return to the computer (Disk 2, Lesson 3, Objective 3b) for further practice and drill if necessary. Then say:
Open End- Based on principles we've learned today could we multiply $\sqrt{6x}$ $\sqrt{3x}$? Call on students to prove this can be done.
Hint: Use same method for proving $\sqrt{a}$ can be multiplied by $\sqrt{b}$.
Discuss possible answers.

Application: multiply-

1. a) $\sqrt{7}$ $\sqrt{3}$ b) $\sqrt{2x}$ $\sqrt{2}$ c) $(\sqrt{a})^2$

2. Find the product of $\sqrt{5x}$ $\sqrt{6x^4}$; $\sqrt{p}$ $\sqrt{r}$ $\sqrt{pr}$

3. Multiply and simplify a) $4\sqrt{3} \times 2\sqrt{3}$ b) $x\sqrt{7}$ $2x\sqrt{7}$

   c) $-3\sqrt{p}$ $4\sqrt{p}$ d) $7\sqrt{x^2}$ $3\sqrt{xy}$

Summary:

1. If the area of a square is x^4 , what is the length of its side?

2. What are the steps involved in multiplying 2 monomial radicals?

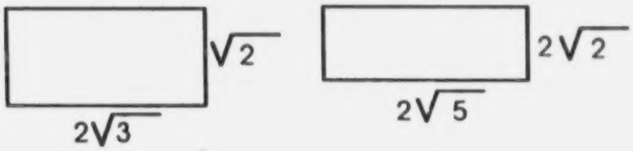
3. What principle of mathematics allows us to multiply radicals?

Homework:

1. a) $\sqrt{3}$ $\sqrt{7}$ b) $2x\sqrt{5}$ $3x\sqrt{6}$ c) $p\sqrt{m} \times 2p\sqrt{m^2}$
   d) $3\sqrt{4} \times 2\sqrt{25}$ e) $-3x\sqrt{6}$ $2y\sqrt{2}$

2. Find the area of a square whose side is: a) $\sqrt{6}$ b) $2\sqrt{5}$

3. a) Find the area of the figure in simplest radical form,
b) Approximate the length of the side correctly to the nearest tenth.


$\sqrt{2}$
$2\sqrt{3}$
$2\sqrt{2}$
$2\sqrt{5}$

TOPIC 41: Simplification of Radicals

Aim: How do we simplify square roots?

Performance Objectives: Students will be able to:

. simplify radicals which do not contain perfect squares.

Vocabulary: radicand, square root, index

Materials:

Hardware: Apple IIe
Software: Elementary Algebra: Exponents Roots & Radicals by Control Data Publishing, Radical Expression: Addition Objective 4b

Motivation:

Place the following example on the board: " $\sqrt{4}$ x $\sqrt{25}$." Ask the pupils to try to determine what the product would be.

Development:

Question: What do these examples illustrate? Answer: The square root of a product of a nonnegative number is equal to the product of the square roots of the numbers.

Let's see how we can use this rule to simplify radicals (flip on related program). Allow students to watch the illustrations provided by this material. Encourage the pupils to try to solve the first practice problem provided by the computer themselves.

After the teacher is certain that all students have had an opportunity to view the illustrative example, place the problem $\sqrt{54}$. How would we go about solving this problem? How did the computer program go about solving this particular example? What was the first step that the computer did? What must we ask ourselves in this step? (find the factors, i.e., $\sqrt{500} = \sqrt{100}$ x $\sqrt{5}$) What should we do next? ($\sqrt{100} = 10$) What is our final answer? ($10\sqrt{5}$) Is that what the computer had?

Have the students work the second example on the computer in the actual program. Afterwards ask: What do we find first? Second? Does this fit with the steps and the answer that the computer gave? Have the students work 3 or 4 practice problems provided by this program. Then briefly review the answers.

Chalkboard Applications:

a) $\sqrt{50}$ b) $\sqrt{\frac{1}{36}}$ c) $\sqrt{x^6 y^2}$

Summary:

In order to simplify a radical what must we do first? (Look for a perfect square as a factor).. then what? Take square root of that factor, and multiply the answer by the factor that is not a perfect square.

Homework:

1. Simplify the expression $\sqrt{72}$, $\sqrt{96}$, $2\sqrt{63}$

2. Which expression is equivalent to $8\sqrt{5}$
   a) $\sqrt{126}$ b) $\sqrt{25}$ c) $\sqrt{320}$ d) $\sqrt{40}$

3. Simplify $\sqrt{\frac{25}{100}}$, $\sqrt{x^2 y^4}$, $\sqrt{a^3}$, $\sqrt{m^6 p^9}$

4. Simplify $\sqrt{36x^4 y^6}$, $\sqrt{40p^6 m^3}$

TOPIC 42: Adding and Subtracting Simple Radicals

Aim: How do we add and subtract radicals?

Performance Objective: Students will be able to:

. combine like radicals.

Materials:

Chalkboard, chalk
Hardware: Large monitor hooked up to an Apple IIe computer.
Software: Practical Algebra - Real Number System by Intellectual Software, L(50) from the menu on the software.
Elementary Algebra: Exponents & Roots, Radicals by Control Data Publishing, choices J & K

Motivation:

1. Add like monomials: $6x + 5x$
2. Explain the rule you used.
3. How would we combine $3\sqrt{X} + 2\sqrt{Y} - 4\sqrt{X}$?

Development:

1. Use of software (Intellectual software L50). Very good descriptions are given.
2. Terms with radical signs are like terms if the radicals are identical and may be combined.
3. Those with simplified unlike radicals are not like terms.
4. Exercises follow that combine like terms. Several examples:
   a. $5\sqrt{2} - 3\sqrt{2} =$
   b. $-2\sqrt{3} + 5\sqrt{7} =$
5. There are exercises that ask if the terms are like or unlike.
   Note: You can add or subtract radicals in the same way you add or subtract like terms.

Applications:

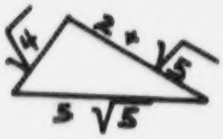
1. $6\sqrt{10} + 8\sqrt{10} - 3\sqrt{10} =$
2. $5\sqrt{8} - 3\sqrt{5} + 3\sqrt{8}$
3. Simplify:

a) $5\sqrt{2} + 2\sqrt{2}$ d) $5\sqrt{7} - 4\sqrt{7}$
b) $9\sqrt{2} - \sqrt{2}$ e) $4\sqrt{3} + 2\sqrt{3} - 5\sqrt{3}$
c) $8\sqrt{3}x - 3\sqrt{3}x$ f) $\sqrt{x} + 5\sqrt{x}$

4. Simplify, if possible:

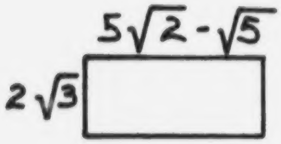
$4\sqrt{3} + \sqrt{2} - 4\sqrt{2}$

5. Find the perimeter of the triangle.

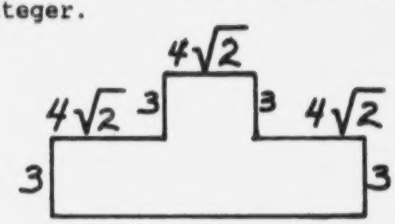


6. Express the perimeter and area of each figure first in radical form and then to the nearest integer.

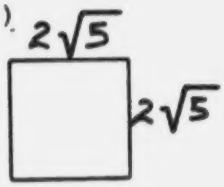
a)



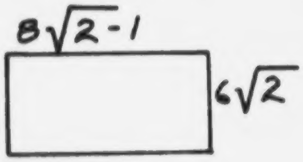
b)



c)



7. Find the area in radical form.



Summary:

1. Does $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ for any nonnegative value given to a and b?

2. How do we combine like radicals?

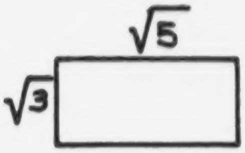
3. How do we combine unlike radicals?

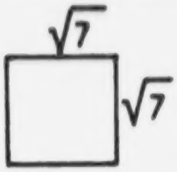
Homework:

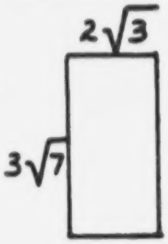
1. Find the sum of: a) $8\sqrt{3} + 5\sqrt{3}$ b) $8\sqrt{3} - 5\sqrt{3}$ c) $5\sqrt{7} - 4\sqrt{7}$

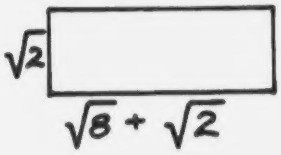
d) $\sqrt{2} + 3\sqrt{2}$ e) $5\sqrt{2} - 7\sqrt{2}$

2. a) Find the perimeter of the figures in radical form.
   b) Approximate the perimeter correct to the nearest tenth.
   c) Find the area in radical form.
   d) Find the area correct to the nearest tenth.









3. Decide whether or not the following can be the lengths of the sides of a triangle. Remember that the longest side must be less than the sum of the other two sides.

   a) $\sqrt{3}$, $2\sqrt{7}$, $\sqrt{12}$
   b) $\sqrt{3}$, $\sqrt{5}$, $\sqrt{11}$
   c) $\sqrt{24}$, $\sqrt{12}$, $\sqrt{12}$
   d) $\sqrt{.24}$, $\sqrt{.12}$, $\sqrt{.12}$
   e) $\sqrt{3}$, $\sqrt{3}$, $\sqrt{13}$
   f) $\sqrt{2}$, $\sqrt{3}$, $\sqrt{8}$
   g) $\sqrt{6}$, $\sqrt{7}$, $\sqrt{29}$
   h) $\sqrt{10}$, $\sqrt{15}$, $\sqrt{40}$

4. Solve completely:

   a) $x^2 = 25$
   b) $x^2 = 24$

5. Multiply $\sqrt{2}(\sqrt{18} - 3\sqrt{2})$ and simplify.

TOPIC 43: Adding and Subtracting Radicals (which must be simplified first)

Aim: How do we add/subtract radicals which have to be simplified first?

Performance Objective: Students will be able to:

- combine unlike radicals by first simplifying each radical and then combining the like radicals.

Materials:

Chalkboard, chalk
Hardware: A large monitor attached to an Apple IIe computer.
Software: Intellectual software - Practical Algebra - Manual (L-55)
Plato software: Exponents, Roots, and Radicals - choice J & K.

Motivation:

Combine: $5\sqrt{2} - 3\sqrt{2} + 2\sqrt{5}$

Explain the rule you used.

Development:

1. Discuss motivation: for the solution we simplify using the distributive property:

   a. $5\sqrt{2} - 3\sqrt{2} + 2\sqrt{5}$

   b. $(5-3)\sqrt{2} + 2\sqrt{5}$ Recall definition!

   c. $2\sqrt{2} + 2\sqrt{5}$

2. Use of software with large monitor. Advance to problem section on L55. Note: Sometimes you have to simplify radicals before you can combine them.

   a. $\sqrt{12} + 5\sqrt{3} =$

   b. It shows simplification $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

   c. Go back: $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

Applications:

1. $3\sqrt{2} + \sqrt{50} =$
2. $\sqrt{20} + (-6\sqrt{5}) =$
3. If an incorrect response is given, program allows for another chance. Students should be encouraged to work at the chalkboard and input their responses into the computer.

4. At the start of the program you can put in the class' code name. For mastery use the Plato software.

5. Simplify:

a) $7\sqrt{3} - 4\sqrt{12}$

b) $7\sqrt{3}x - \sqrt{12}x$

c) $7x\sqrt{5xy^2} - \sqrt{20x^3y^4}$

d) $\sqrt{75} - \sqrt{27} + \sqrt{12}$

e) $\sqrt{8y^3} - \sqrt{2y^3}$

6. Simplify, if possible:

a) $4\sqrt{3} + \sqrt{2} - 4\sqrt{2}$

b) $\sqrt{22} + \sqrt{11} - \sqrt{66}$

Homework:

1. Simplify:

a) $\sqrt{a} + 8\sqrt{a}$

b) $5\sqrt{3} + \sqrt{12}$

c) $\sqrt{72x^3} - \sqrt{18x^3}$

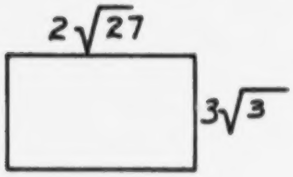
d) $\sqrt{18} - 2\sqrt{2}$

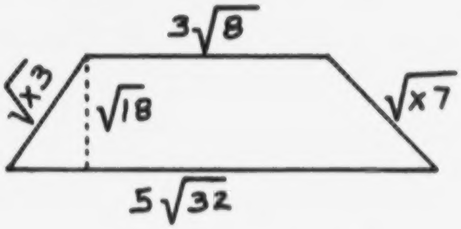
e) $4\sqrt{5} + 7\sqrt{2} - 7\sqrt{5}$

f) $\sqrt{50} + 3\sqrt{2}$

2. Find the value of the expression $\sqrt{18} - \sqrt{2}$.

3. Find the area and perimeter and simplify your answer:





TOPIC 44: Pythagorean Theorem

Aim: How do we prove and apply the Pythagorean Theorem?

Performance Objectives: Students will be able to:

- state and prove informally the Pythagorean theorem
- define Pythagorean triples.
- solve numerical and algebraic exercises concerning sides of a right triangle by applying the Pythagorean theorem.
- find the square and square root of real numbers.
- express an irrational number in simplest radical form.
- apply the Pythagorean theorem in solving exercises such as computing the length of:
  - a. altitude of an equilateral or isosceles triangle
  - b. side of a rhombus given the diagonals
  - c. the altitude of an isosceles trapezoid
- form the converse of the Pythagorean theorem.
- determine if the triangle is a right triangle, given the lengths of the three sides.

Vocabulary: Pythagorean theorem, Pythagorean triples, radical form, rhombus, isosceles trapezoid, converse

Materials:

Apple computer lab
software: Special Topics in Mathematics: Powers and Roots by Intellectual Software, Apple II)

Motivation:

A doorway is 8 feet high and 6 feet wide. What is the widest rectangular panel that can be taken throught the doorway? ("slightly" less than 10 feet)

Development:

1. In order to prove the Pythagorean theorem, consider the following right triangle ABC, with altitude CD.

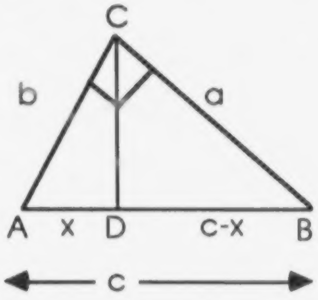


figure 1

a. Prove: $a^2 + b^2 = c^2$ (in a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.)

b. Use the theorem from previous learnings for each leg:

$$\frac{c}{a} = \frac{a}{c-x} \qquad a = c + cx$$

$$a^2 = c^2 - cx$$

$$\frac{c}{b} = \frac{b}{x} \qquad b = cx$$

$$b^2 = cx$$

$$a^2 + b^2 = c^2 - cx + cx$$

$$a^2 + b^2 = c^2$$

2. A Pythagorean triple is a set of three integers (a, b, c) which could represent the lengths of the sides of a right triangle. That is, they satisfy the equation.

   a. $a^2 + b^2 = c^2$.

   For example, the following are Pythagorean triples: (3,4,5), (5,12,13), and (8,15,17). Note that if each number in the Pythagorean triple is multiplied by positive integer x, then another Pythagorean triple is formed for a similar right triangle). For example, (6,8,10) and (10,24,26) are Pythagorean triples.

3. Irrational numbers should be expressed in simplest radical form by factoring out any perfect squares. For example, $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$. To the nearest tenth, $3\sqrt{2} = 4.2$.

4. The following are numerical and algebraic exercises (first express the answers in simplest radical form and then to the nearest tenth).

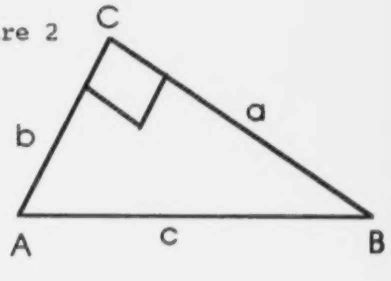
   a. Given: a = 24, b = 45
   Find: c (the sides are three times the Pythagorean triple (8,15,17). c = 51

   b. Given: a = 7, b = 9
   Find: c ($\sqrt{130}$ or 11.4, to the nearest tenth)

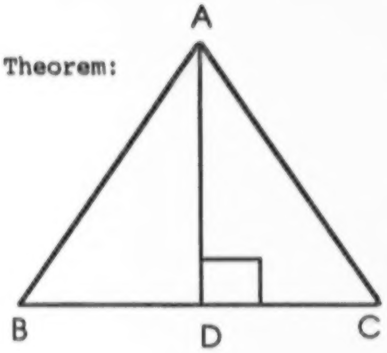
   c. Given: b = 4, c = 8
   Find: a (4 $\sqrt{3}$ or 6.9 to the nearest tenth.)

   d. Given: a = x, B = x + 2, C = x + 4
   Find: x (6)

figure 2



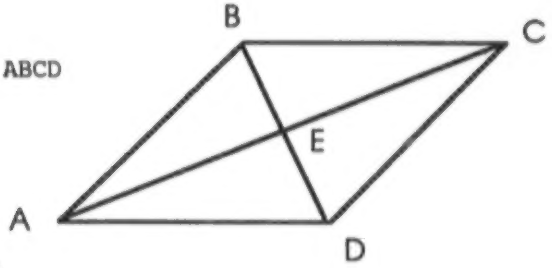
5. The following are applications of the Pythagorean Theorem:



a. Given: AB = 9, AC = 9 figure 3
altitude $\overline{AD}$, BC = 12
Find: AD
(BD = 6, AD = $\sqrt{45}$ or approximately 6.7)

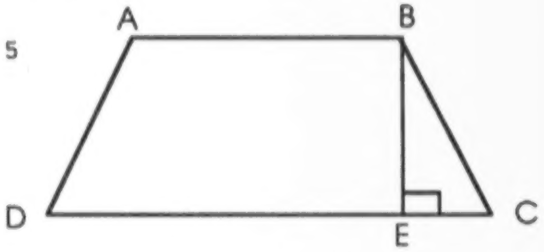
b. Given: Rhombus ABCD

figure 4



BD = 30, AC = 16
Find: BC (the diagonals of a rhombus are perpendicular to each other.
BE = 15, EC = 8, BC = 17.)

figure 5



c. Given: Isosceles trapezoid
ABCD with $\overline{AB}$ // $\overline{CD}$, altitude
BE, AD = BC, AB = 12, DC = 20,
BC = 5
Find: BE (draw $\overline{AG} \perp \overline{DC}$, GE = 12
EC = 4, BE = 3)

6. The converse of the Pythagorean theorem which is also a theorem is: If the square of the third side equals the squares of the other two sides, then the triangle is a right triangle. Given the lengths of the three sides of a triangle, the converse can be used to determine if the triangle is a right triangle. For example, indicate which of the following triangles with the given side lengths are right triangles.

a. 7, 12, 14 (no, since $7^2 + 12^2 \neq 14^2$

b. 40, 85, 75 (yes)

c. $\sqrt{5}$, 1, 2 (yes, since $(\sqrt{5})^2 = 1^2 + 2^2$

d. $\sqrt{6}$, $\sqrt{23}$, 4 (no)

7. Use the computer for practice and reinforcement of the concepts learned. Load the program SPECIAL TOPICS IN MATHEMATICS: Powers and Roots (Intellectual Software, Apple II). The mathematical processes of square root are explained in this interactive program with reinforcement by drill and practice exercises. Additional remediation is given after each incorrect answer. The topics covered include square roots, the radical sign, the arithmetic method for finding square roots, non-perfect square roots.

Applications

If you are asked to find the length of the side of a polygon and if there is a right angle in the diagram, then consider applying the Pythagorean Theorem. For example:

1. In rectangle ABCD, AB = x, BC = X + 7, and diagonal BD = X + 8. Find BD.
    a. ($x^2 - 2x - 15 = 0$, $x = 5$, BC = 13)
2. The perimeter of a rectangle is 28 centimeters. If the length of a diagonal of the rectangle is 10 cm., find the number of centimeters in the length and width of the rectangle. (6, 8)
3. What is the length of a diagonal of a square of side length 2? ($2\sqrt{2}$)

Summary:

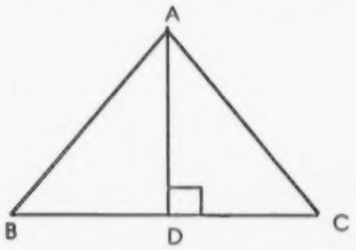
1. What is the Pythagorean theorem?
2. What is a Pythagorean triple?
3. State examples of Pythagorean triples.
4. How do you express an irrational number in simplest form?
5. What is the converse of the Pythagorean theorem?

Homework:

Express all answers first in simplest radical form and then to the nearest tenth.

1. The diagonals of a rhombus have lengths of 8 and 6. What is the length of a side of the rhombus?
2. In rectangle ABCD, BC = 12 and CD = 16. Find BD.
3. What is the perimeter of a square which has a diagonal of length 12?
4. In right triangle ABC, BC = x, AC = 8 - x, and hypotenuse AB = 6. Find BC. (Note to teacher: the quadratic formula is required to solve the equation.)
5. Given: equilateral triangle ABC, altitude $\overline{AD}$, AB = 14, Find $\overline{AD}$.

figure 6



6. Given: isosceles trapezoid ABCD, AB // DC, AD = BC, altitude BE, BE = 9, AB = 16, DC = 22
Find: BC

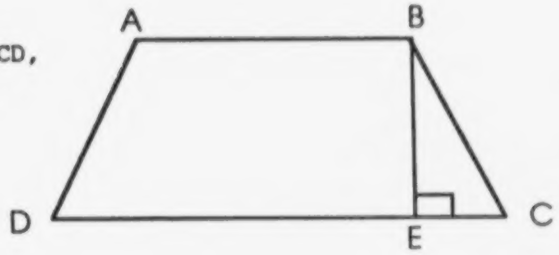


figure 7

7. Can a triangle with side lengths of $x + 1$, $5x$ and $5x + 1$ be a right triangle?

Solutions:

1. 5

2. 20

3. $24\sqrt{2}$ or 33.9

4. $x^2 - 8x + 14 = 0$

   $x = \frac{8 - \sqrt{8}}{2}$ $\qquad$ $x = \frac{8 + \sqrt{8}}{2}$

   $x\ 4 - \sqrt{2}$ or 2.6 $\qquad$ $x\ 4 + \sqrt{2}$ or 5.4

5. $x = \sqrt{147} = 7\sqrt{3} = 12.1$

6. $BC = \sqrt{90} = 3\sqrt{10}$ or 9.5

7. yes, $x = 8$

   The sides are 9, 40, and 41.

TOPIC 45: Multiplying Two Binomials - I

Aim: How do we compute the product of two binomials?

Performance Objectives: Students will be able to:

- write the product of two binomials with at most two variables.
- form the product by multiplying either horizontally, or vertically.

Vocabulary: review: binomial, trinomial, product, horizontal, vertical

Materials:

Software: Binomial Multiplication by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers, Monitor and Disk Drive

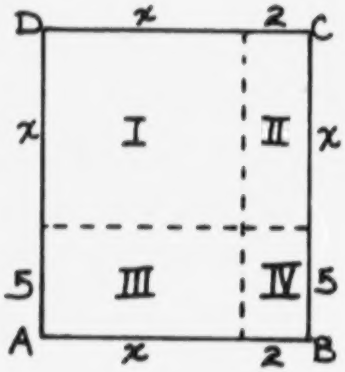
Motivation:

> The length of a rectangle is x + 5 and the width is x + 2. Express the area in terms of x.

Development:

1. Refer to motivation. Using the formula, we write:

Area = (length)(width)
A = (x + 5)(x + 2)



Have students multiply (x + 5) (x + 2) using the associative and distributive properties.

$$
\begin{aligned}
(x + 5)(x + 2) &= (x + 5)(x) + (x + 5)(+2) && \text{Why?}\\
&= x^2 + 5x + 2x + 10 && \text{Why?}\\
&= x^2 + (5x + 2x) + 10 && \text{Why?}\\
&= x^2 + (5 + 2)x + 10 && \text{Why?}\\
&= x^2 + 7x + 10
\end{aligned}
$$

Have students notice that the distributive property of multiplication over addition is used several times. Consider the product of (x + 2)(x + 5), will it yield the same result as above? Why?

2. Let's use the computer to do problems of this type in which you have to find the product of 2 binomials.

3. Insert the Binomial Multiplication disk into the disk drive and turn on the monitor and computer.

4. From the Main Menu, select 1 for Instructions.

5. Read the instructions and select 2 from the Menu.

6. This will show you how to multiply two binomials using the FOIL method. Note that the First and Outer products are computed for you.

7. What is the product of the Inside Terms? (-2xy)

8. What is the product of the Last Terms? (-14y)

9. After you finish multiplying, what should you do? (simplify the product)

10. Read the description of why the FOIL method works.

11. Notice that FOIL is just a shortcut for the distributive and commutative laws.

12. Follow the extra practice session on using the FOIL method.

13. Exit the program and turn off the monitor and disk drive.

Summary:

1. Find an example when the product of two binomials is a trinomial.

2. Find an example when the product of two binomials is a binomial.

3. Find an example when the product of two binomials is a polynomial of four terms.

Homework:

1. Express the product $(x + 2)(x - 5)$ as a trinomial.

2. Express the product $(x - 3)(x + 3)$ as a binomial.

3. Express as a trinomial $(3x - 1)(x + 2)$.

4. Compute the products for the following using

a) horizontal form	b) vertical form
$(x - 2)(x + 6)$	$(3a + 2)(4a - 1)$
$(7a + 2)^2$	$(4x + y)(5x - 2y)$

5. Compute the products for the following using distributivity and check by giving the variables a numerical value:
   - a) $(2a - 7)(3a + 4)$
   - b) $(3b + 4)(2b - 5)$
   - c) $(7c + 2a)(5c - 2a)$
   - d) $(3d - 2e)(3d + 2e)$
   - e) $(6g + 9h)(6g - 9h)$
   - f) $(2\frac{1}{2} + 1)(2\frac{1}{2} - 1)$
   - g) $(2.5r + 3s)(2.5r - 3s)$
   - h) $(2x - 3y)(2x + 3y)$

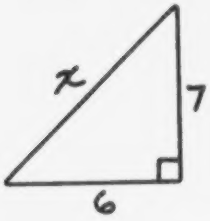
6. Compute the products using the vertical format and check by giving the variables a numerical value:
   - a) $(3s + 7t)(2s - 3t)$
   - b) $(5x + 2y)(5x - 3y)$
   - c) $(6u - 7v)(6u + 7v)$

7. Show by means of areas of rectangles that:

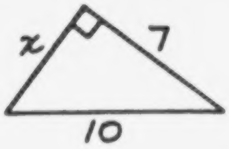
$(a + b)(a - b) = a^2 - b^2$. Assume a > b.

8. Compute to the nearest tenth the length of the missing side.

a)



b)



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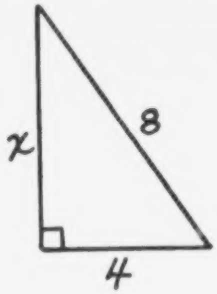


TOPIC 46: Multiplying Two Binomials - II

Aim: How do we multiply two binomials mentally?

Performance Objectives: Students will be able to:

- write the product of two binomials using the FOIL method.
- solve problems using multiplication of two binomials.

Materials:

Software: Binomial Multiplication by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers, Monitor and Disk Drive

Motivation

Find each product:

1. $(x + 3)(x - 3)$
2. $(2x + 5)(2x - 5)$
3. $(x - 2)(x - 2)$
4. $(a + 4)(a + 4)$
5. $(x - 4)(x - 3)$

Which of these products are perfect square trinomials?

Development:

1. Discuss motivation problem. Products 3 and 4 are perfect square trinomials (these problems may be interpreted as finding the area of a square, since the two factors are equal). Product 5 is a trinomial but not a perfect square. Products 1 and 2 are binomials which are each the difference of two squares. Ask, "How can you recognize a product of two binomials such as this?" Students should observe that the factors are a sum and a difference of the same terms. Why is there no "middle term" in this case? Two of the partial products are opposites and add up to zero.

2. Insert the Binomial Multiplication disk into the disk drive and turn on the monitor and the computer.

3. From the Menu choose 3 for problems.

4. You will be asked how many problems you want to do. Select 9.

5. Do the problems using the FOIL method as outlined.

6. Make sure you simplify the products.

7. Examine the error summary for each problem.

8. Examine the Total Error Analysis after you finish all the problems.

9. Exit the program and turn off the monitor and computer.

Summary:

1. What do the letters in FOIL represent?

2. Explain why the product of two binomials will sometimes have only two terms.

3. Using the FOIL method, state each product:
   $(a + b)(c + d)$ $(a + b)(a - b)$ $(a + b)(a + b)$

Homework:

1. In the last lesson homework examples were solved by horizontal and vertical method. Multiply again using FOIL method.
   $(x + 2)(x - 5)$
   $(x - 3)(x + 3)$
   $(3x - 1)(x + 2)$
   $(3a + 2)(4a - 1)$
   $(7a + 2)^2$
   $(x - 2)(x + 6)$

2. The length of a rectangle is represented by $x + 3$ and the width by $x + 8$. Express the area of the rectangle as a trinomial in terms of x.

3. Find the area of a square whose side is $2x - 7$.

4. Simplify: $(x - 3)(x - 2) + (x - 2)(x + 5)$

5. Use the FOIL method to compute the following products and check by giving each variable a numerical value.
   a) $(2a + 3b)(2a - 3b)$
   b) $(4c - d)(4c + d)$
   c) $(7g + 3h)(3g + 7h)$
   d) $(3x - 2y)(3x - 2y)$
   e) $(4u + 3v)^2$
   f) $(2.5 - x)(7.5 + x)$

6. Solve to the nearest integer.
   a) $x^2 = 200$
   b) $y^2 = 800$
   c) $r^2 = 300$
   d) $s^2 = 1200$
   e) $u^2 - 50 = 0$
   f) $v^2 - 5 = 0$
   g) $w^2 - 12.4 = 0$
   h) $z^2 - 30.4 = 0$

7. Find two binomials whose computed product is:
   a) $x^2 - 9$
   b) $y^2 - 1$
   c) $4x^2 - 1$
   d) $4r^2 - 9$
   e) $a^2 - b^2$
   f) $4a^2 - 9b^2$
   g) $a^2 + 2a + 1$
   h) $a^2 + 2ab + b^2$

TOPIC 47: Factoring

Aim: What is meant by factoring?

Performance Objectives: Students will be able to:

- define factoring.
- use the word factoring correctly in describing expressions.
- factor algebraic expressions using greatest common factor.
- find the prime factors of integers.

Vocabulary: product, factors, factoring, distributive property of multiplication over addition

Materials:

Software:	Factoring Algebraic Expressions by Microcomputer Workshops Courseware
Hardware:	Apple II, IIe, IIc Computers, Monitor and Disk Drive

Motivation:

> A rectangle has an area of 24 cm^2. What are some possible dimensions for the rectangle?

Development:

1. Elicit pairs of numbers which could serve as length and width:
   24,1   8,3   12,2   6,4
   If students do not suggest noninteger dimensions such as 5,4.8 or 10,2.4 for the rectangle, elicit this idea by asking, "Could the rectangle have a length equal to 10cm? Could you have a square with this area?" Ask, "How many different rectangles could be formed having an area equal to $24cm^2$?" (infinitely many) Ask, "How many different rectangles with integer dimensions have this area? (only the first four listed above) The integers 1,2,3,4,6,8,12,24 are called the factors of 24. The process of finding such numbers (or, as we shall see later, variables) is called factoring.

2. What are the factors of 48? List all pairs of positive integers whose product is 48. Examine one set of factors, i.e., 48 = 12 x 4. Can 12 and 4 be factored? Since 12 = 6 x 2 and 4 = 2 x 2 then 48 = 6 x 2 x 2 x 2. Which of these factors can be factored still further?

3. Demonstrate that the same techniques can be applied to algebraic expressions, i.e., $6x^2y^3 = 3 . 2 x x y y y$

   $9xy^5 = 3 . 3 x y y y y y$

   What is the greatest common factor of $6x^2y$ and $9xy^5$? ($3xyyy$ or $3xy^3$)
   Students should observe that the coefficient of the GCF is the GCF of

the coefficients, and that each variable appears in the GCF with the smallest exponent shown in either term.
Ask: "What is the GCF of $3x$ and $9y$? (3) Can the expression $3x + 9y$ be factored?" (Yes, as $3(x + 3y)$.) This may be checked by multiplying, using the fact that multiplication distributes over addition.) Have the class describe the procedure for factoring polynomials of more than one term: find the GCF of all terms (this is the first factor); divide the given polynomial by the GCF (this is the second factor). Apply to other algebraic expressions such as $4x - 6y$, $3x^2 - 6x^3 + 12x$.

4. Insert the Factoring Algebraic Expressions disk into the disk drive and turn on the monitor and computer.

5. Type your name and when it asks if you want instructions, press N.

6. When asked what kind of problems you want, type 4 for Common Factor. When asked how many problems you would like type 6.

7. Factor each expression using the arrow keys to move the cursor and the spacebar to erase.

8. When you are finished press RETURN.

9. Look at the Error Summary for each problem you do.

10. Examine the Total Error Analysis to see where you had difficulty.

Summary:

1. What process is followed when you are asked to factor an integer? (Factoring is restating a given expression as an equivalent product.)

2. Explain and illustrate GCF.

Homework:

1. Compute the following products:

   a) $3a(2a + 3b)$
   b) $4c^2(3 - 5c)$
   c) $(8 - 3)(8 + 3)$
   d) $2x(x^2 + x + 1)$
   e) $3ac^2(4bc - 2ab)$
   f) $(5\tfrac{2}{3} + 4\tfrac{1}{3})(5\tfrac{2}{3} - 4\tfrac{1}{3})$

2. Factor, showing the greatest common factor as one factor:

   a) $3a + 6b$
   b) $3a + 6a^2$
   c) $4 - 12x$
   d) $x^3 + x^2 + x$
   e) $6a^2c(4bc - 2ab)$
   f) $12(r + s) - 7t(r + s)$

3. Compute two ways, one way by factoring:
   a) 13 x 7 + 13 x 3

   b) 3.1416 x 76 + 3.1416 x 24

   c) $7 \times 2\frac{3}{4} + 7 \times 7\frac{1}{4}$

   d) 9.36 x 43 - 9.36 x 33

4. Solve:

   a) 2a + 3a = 30
   b) 7x - 3x = 12
   c) 2a + 3a = 30
   d) 7x - 3x = 12

TOPIC 48: Factoring the Difference of Two Squares

Aim: How do we factor expressions of the form $x^2 - a^2$?

Performance Objectives: Students will be able to:

- multiply mentally $(x + a)(x - a)$.
- recognize an algebraic expression which is the difference of two perfect squares.
- factor a difference of two perfect squares.
- recognize that $x^2 + a^2$ is not factorable (with real coefficients).

Vocabulary: perfect square, difference

Materials:

Software: Factoring Algebraic Expressions by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers, Monitor and Disk Drive

Motivation:

Which (if any) of the following expressions are equivalent to $x^2 - 9$?

a) $x(x - 9)$
b) $3(x - 3)$
c) $(x - 3)(x - 3)$
d) $(x - 3)^2$
e) $(x + 3)(x - 3)$

Development:

1. Discuss motivation. Note that both terms of $x^2 - 9$ are perfect squares and we are taking the difference of these two squares. Demonstrate that only $(x + 3)(x - 3)$ is equivalent. When we say that $x^2 - 9 = (x + 3)(x - 3)$, we have factored the expression $x^2 - 9$ by writing it as the indicated product of expressions which have lower degree. This type of factoring is not like factoring out the GCF. Students should observe that the terms in the expression $x^2 - 9$ have no common factor except 1 (they are relatively prime).

   Elicit the generalization: Any difference of two perfect squares may be factored by this rule: $A^2 - B^2 = (A + B)(A - B)$.

2. Let's use the computer to apply this rule in factoring the difference of two perfect squares.

3. Insert the Factoring Algebraic Expressions Disk and turn on the monitor and computer.

4. Type your name and N for no when asked if you want instructions.

5. When asked what kind of problems you want, TYPE 3 for Difference of 2 Squares.

6. When asked how many problems you would like type 9.

7. Factor each expression using the arrow keys to move the cursor and the spacebar to erase.

8. When you are finished press RETURN.

9. Look at the Error Summary for each problem you do.

10. Examine the Total Error Analysis to see where you had difficulty.

11. Remove diskettes from the computers and place them in their protective covers. Turn off the computers and monitors.

Summary:

1. Factor each expression and explain the difference in strategy used for each:

   $25x^2 - 10y^2$ $\quad\quad$ $25x^2 - 49y^2$

2. Factor $50x^6 - 18y^2$

Homework:

1. Compute the product using "multiplication distributing over addition and subtraction":

   a) $8 \times 3\frac{1}{4}$

   b) $6 \times 4\frac{2}{3}$

   c) $12 \times (2\frac{3}{4} - 1\frac{1}{3})$

   d) $30 \times (4\frac{3}{5} + 1\frac{1}{4} - 4\frac{2}{3})$

2. Compute the products:

   a) $(a + b)(a - b)$
   b) $(a + b)(a + b)$
   c) $(3a + 1)(3a - 1)$
   d) $(a - b)(a - b)$
   e) $(1 - 4x)(1 + 4x)$
   f) $(1 + 4x)(1 + 4x)$

3. Factor using the difference of squares and compute:

   a) $8^2 - 7^2$

   b) $51^2 - 49^2$

   c) $1.3^2 - .7^2$

   d) $(5\frac{2}{3})^2 - (4\frac{1}{3})^2$

   e) $111^2 - 89^2$

   f) $10.5^2 - 9.5^2$

4. Factor:

a) $r^2 - s^2$

b) $1 - a^2$

c) $b^2 - 1$

d) $x^2 - 9$

e) $16 - y^2$

f) $4a^2 - 1$

g) $1 - 4c^2$

h) $9x^2 - 4y^2$

i) $(a + b)^2 - (a - b)^2$

5. Factor:

a) $a^2 - \frac{1}{c^2}$

b) $\frac{r^2}{s^2} - \frac{s^2}{r^2}$

c) $x^4 - 1$

d) $y^4 - 16$

TOPIC 49: Trinomial Factoring - I

Aim: How do we factor an expression of the form $x^2 + bx + c$?

Performance Objectives: Students will be able to:

- factor an expression of the form $ax^2 + bx + c$ where $a = 1$.
- state the significance of the values and signs of "b" and "c".
- check the solution.

Vocabulary: trinomial, coefficient, term

Materials:

Software:	Factoring Algebraic Expressions by Microcomputer Workshops Courseware
Hardware:	Apple II, IIe, IIc Computers, Monitor and Disk Drive

Motivation:

Multiply:

1. $(x + 5)(x + 3)$
2. $(x + 5)(x - 3)$
3. $(x - 5)(x + 3)$
4. $(x - 5)(x - 3)$

What similarities and differences do you notice in these products?

Development:

1. Discuss motivation. Elicit that: In every case, the product is a trinomial. In every case, the absolute value of the constant term is 15, which is the product of 3 and 5. When the signs in the factors are the same, the constant term is positive; when the signs in the factors are different, the constant term is negative. In every case, the coefficient of the middle term is the sum of the constant terms in the factors.

2. These observations will help us to factor trinomials such as $x^2 + 5x + 6$. Note (1) there is no common factor and (2) this is not a "difference of two squares". The answer will be in the form $(x + p)(x + q)$. The product pq must equal 6. Ask: "What are the possible ways to factor 6, using integer factors?" (6 x 1, 2 x 3, -2 x -3, -6 x-1)

   Since the constant term in the trinomial is positive six, we know that p and q must have the same sign. All of these products include a constant term of +6:

   $(x + 6)(x + 1)$　　$(x + 3)(x + 2)$
   $(x - 6)(x - 1)$　　$(x - 3)(x - 2)$

   Ask: "Which has +5x as the middle term?" (Find each product, using the FOIL method.) Elicit that $(x + 3)(x + 2)$ is the only correct factorization of $x^2 + 5x + 6$.

3. Let's use the computer to factor other trinomials of this type ($x^2 + bx + c$)

4. Insert the Factoring Algebraic Expressions disk into the disk drive and turn on the monitor and computer.

5. Type in your name and N when asked if you want instructions.

6. Type 1 when asked what kind of problems you want.

7. Type 9 when asked how many problems you would like.

8. Factor each trinomial using the arrow keys to move the cursor and the spacebar to erase.

9. Be careful when typing in your + and - signs.

10. Press RETURN when finished with each problem.

11. Look at the Error Summary for each problem.

12. Examine your Total Error Analysis.

13. Press 6 to exit the program.

14. Turn off the monitor and computer and remove the disk.

Summary:

1. What is the first part of the trinomial you should examine to gain information about its factors? (the constant term)

2. What do all the trinomials considered today have in common? (Note in all cases here the coefficient $a = 1$.)

Homework:

1. Compute the product using the FOIL method and check by assigning a small number to the variable.

   a) $(a + 1)(a + 2)$
   b) $(a - 1)(a - 2)$
   c) $(a - 1)(a + 2)$
   d) $(a + 1)(a - 2)$
   e) $(2c + 1)(2c + 1)$
   f) $(1 - 2c)(1 - 2c)$
   g) $(2c - 3)(2c + 1)$
   h) $(3x - 5)(2x + 4)$

2. Express each trinomial as a product of two binomials (factor):

   a) $a^2 + a - 2$
   b) $b^2 + 3b + 2$
   c) $c^2 - 3c + 2$
   d) $d^2 - d - 2$
   e) $e^2 + 7e + 6$
   f) $f^2 - 7f + 6$
   g) $g^2 + 5g - 6$
   h) $h^2 - 5h - 6$

3. **Factor:**

a) $a^2 - 4a$

b) $b^2 + 9b$

c) $c^3 + c^2$

d) $y^2 - y - 12$

e) $r^2 - 4 - 3r$

f) $1 - \frac{1}{a^2}$

g) $1 + \frac{1}{a} - \frac{2}{a^2}$

h) $x^2 - x - 30$

i) $y^2 + 7y - 30$

TOPIC 50: Trinomial Factoring - II

Aim: How do we factor an expression of the form $ax^2 + bx + c$ where $a \neq 1$?

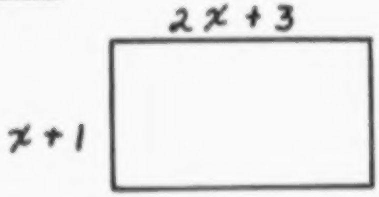
Performance Objectives: Students will be able to...

- factor an expression of the form $ax^2 + bx + c$ where $a \neq 1$.
- state the significance of $a \neq 1$.
- check the solution.

Materials:

Software: Factoring Algebraic Expressions by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers, Monitor and Disk Drive

Motivation:



1. Express the area of this rectangle as a trinomial in terms of x.
2. Factor $2x^2 + 5x + 3$.

Development:

1. Discuss motivation. The students will "see" the solution to problem 2, since this trinomial is the same as the area of the rectangle.

2. Ask: "How does the trinomial we have just factored differ from those factored yesterday? Recall that in the previous lesson, in factoring trinomials such as $x^2 + 5x + 6 = (x + 2)(x + 3)$, the product of 2 and 3 was the coefficient of the middle term (5). Is this true for $2x^2 + 5x + 3 = (2x + 3)(x + 1)$? (The constant term is the product of the constant terms in the factors, but the coefficient of the middle term is not their sum.) Elicit that since $a \neq 1$, the coefficients of the "inner" and "outer" products cannot both match the constants in the factors. The coefficient of the middle term of the product will (continue to) be the sum of these coefficients, which will now be different from the sum of the constants in the factors.

3. To factor trinomials such as the above, establish that combinations of all possible factors of both a and c (in the general form $ax^2 + bx + c$) must be tried to find the combination which produces the correct middle term. The rules for signs are not affected by $a \neq 1$.

4. Let's use the computer to solve other problems of this type ($ax^2 + bx + c$ where $a \neq 1$)

5. Insert the Factoring Algebraic Expressions disk into the disk drive and turn on the monitor and the computer.

6. Type your name and N for no when asked if you want instructions.

7. Type 2 when asked what kind of problems you want.

8. Type 9 when asked how many problems you would like.

9. Factor each expression using the arrow keys to move the cursor and the spacebar to erase.

10. Press RETURN when you finish each problem and look at your Error Summary.

11. Examine your Total Error Analysis to see where you had difficulty.

12. Press 6 to exit the program.

13. Turn off the monitor and the computer and remove the disk.

Summary:

1. Explain why it is more difficult to factor a trinomial of the form $ax^2 + bx + c$ when $a \neq 1$.

2. How would you verify that a trinomial has been factored correctly?

Homework:

1. Compute using the FOIL method and check by distributing:

   a) $(a + 1)(2a + 1)$
   b) $(b + 2)(2b + 1)$
   c) $(2c + 1)(c + 2)$
   d) $(d + 1)(2d - 1)$
   e) $(2e + 1)(2e + 1)$
   f) $(2f + 1)(3f + 1)$
   g) $(3g - 1)(3g + 3)$
   h) $(5h + 2r)(2h - 5r)$

2. Factor:

   a) $a^2 + a - 2$
   b) $2b^2 + b - 1$
   c) $c^2 - 3c + 2$
   d) $2d^2 - 3d + 1$
   e) $2e^2 + e - 6$
   f) $2f^2 - f - 6$
   g) $6g^2 + 7g + 1$
   h) $6g^2 - 5g - 1$
   i) $4a^2 - 4a + 1$
   j) $4b^2 + 1 - 4b$
   k) $4r^2 + 12rs + 9s^2$
   l) $6x^2 + 11xy - 10y^2$

3. Factor (and evaluate when numerical).

a) $52^2 - 48^2$

b) $(7\frac{1}{2})^2 - (6\frac{1}{2})^2$

c) $14.2857 \times 93 + 14.2857 \times 7$

d) $9r^2 - 6r + 1$

e) $9s^2 - 1$

f) $2u^2 - u - 3$

g) $3 - v - 2v^2$

h) $\left(\frac{u + v}{2}\right)^2 - \left(\frac{u - v}{2}\right)^2$

TOPIC 51: Complete Factoring

Aim: Can algebraic expressions be factored more than once?

Performance Objectives: Students will be able to:

- state three types of factoring.
- state the meaning of "factor completely".
- factor expressions completely.

Vocabulary: completely factored

Materials

Software: Factoring Algebraic Expressions by Microcomputer Workshops Courseware
Hardware: Apple II, IIe, IIc Computers, Monitors and Disk Drives

Motivation:

Factor each expression:

1. $25x^2 - 40x$
2. $25x^2 - 49$
3. $5x^2 - 6x + 1$
4. $x^2 + x + 3$

Explain the strategy you used for each example.

Development:

1. Discuss motivation. Elicit that the first two expressions are binomials, each containing the term $25x^2$, yet a different method is used in each.

   Elicit that expressions 3 and 4 are trinomials. Each has a positive constant term, indicating that the constants in the factors will have the same sign (both positive or both negative). The first can be factored as $(5x - 1)(x - 1)$. $x^2 + x + 3$ cannot be factored, since $(x + 3)(x + 1)$ and $(x - 3)(x - 1)$ each produce a "middle term" different from $+1x$.

2. Consider $2x^2 + 4x + 2$. Is it possible to factor this trinomial? In this exercise, students will often suggest $(2x + 2)(x + 1)$ as the answer; others will note the common factor and obtain $2(x^2 + 2x + 1)$. Ask: "Is the problem finished?" Elicit that in each case one factor can be factored; i.e., that the "factoring" process can continue.

$2x^2 + 4x + 2$	$2x^2 + 4x + 2$
$(2x + 2)(x + 1)$	$2(x^2 + 2x + 1)$
$2(x + 1)(x + 1)$	$2(x + 1)(x + 1)$

As with integers, it should be noted here that regardless of the techniques employed, the final "completely factored" form will be the same (except, possibly, for the order of the factors).
Ask: "How do you know when you have factored an expression completely?"
Elicit: When each factor is an expression which cannot itself be factored, such as $x + 1$ or $x^2 + x + 3$. (These may be called "prime polynomials". All monomials are considered "prime" in this regard.)
An analogy may be made with "reducing" fractions and "reducing fractions completely".

3. Ask the students to factor $25x^2 + 30x + 5$. Is it easier to factor out the greatest common factor first or to try to write this expression as the product of two binomials? Students should conclude that since it is easier to factor a trinomial with small coefficients, the best approach is to begin by examining the terms in the hope of finding a common factor; $25x^2 + 30x + 5 \longrightarrow 5(5x^2 + 6x + 1) \longrightarrow 5(5x + 1)(x + 1)$ will generally take less time than routes through $(5x + 1)(5x + 5)$ or $(25x + 5)(x + 1)$ to the same goal.

4. Let's use the computer to completely factor different types of problems.

5. Insert the Factoring Algebraic Expressions disk into the disk drive and turn on the monitor and the computer.

6. Type your name and N for no when asked if you want instructions.

7. Type 9 when asked how many problems you would like.

8. Decide what is the best way to factor each problem. Factor each problem completely using the arrow keys to move the cursor and the spacebar to erase.

9. Press RETURN when you are finished each problem. Look at your Error Summary after each problem.

10. Examine your Total Error Analysis to see where you had difficulty.

11. Press 6 to exit the program and turn off the equipment.

Summary:

1. Given a polynomial to factor, how would you begin?

2. When is the "factoring" process finished?

Homework:

1. Factor completely:

   a) $2r^2 - 2s^2$

   b) $9u^2 - 9v^2$

   c) $9u^2 - 4v^2$

   d) $u^2 - 1$

   e) $ar^2 - as^2$

   f) $2x^2 + 4x + 2$

   g) $3r^2 - 6r + 3$

   h) $5 + 5s - 30s^2$

   i) $4r^2 - 16 - 12r$

   j) $15 + 5u - 10u^2$

   k) $rs^2 - r$

   l) $2x^4 - x^2 - 1$

2. Using factoring, compute:

   a) $9^2 - 8^2$

   b) $27^2 - 24^2$

   c) $3.14 \times 51^2 -$
      3.14×49^2

   d) $(6\frac{3}{4})^2 - (3\frac{1}{4})^2$

   e) $2700^2 - 2300^2$

   f) $30^2 - 2 \times 29 \times 30 + 29^2$

3. If the domain of each variable is the set of integers from -10 to 10 inclusive, solve:

   a) $x^2 = 25$

   b) $4a^2 = 100$

   c) $c^2 + c - 6 = 0$

   d) $x^2 < 9$

   e) $2r^2 + r - 6 = 0$

TOPIC 52: Quadratic Equations - I

Aim: What is a quadratic equation? How do we identify the coefficients of a quadratic equation?

Performance Objectives: Students will be able to:

- define "quadratic equation"
- identify a quadratic equation
- identify the A, B, C coefficients of a quadratic equation

Vocabulary: quadratic equation, standard form

Materials:

Chalkboard, chalk
Hardware: Large monitor attached to an Apple 2E computer
Software: Intellectual software - Quadratic Equations II (L15) Introducing Quadratics

Motivation:

1. How do the equations differ? How are they the same?
   a. $2X^3 + 5X + 3 = 0$
   b. $2X^7 = 0$
   c. $X^2 + 5X - 7 = 0$

Development:

1. Software selection L15 Introducing Quadratics
   a. $AX^2 + BX + C = 0$
   b. identify from $2X^2 - 5X + 7 = 0$ C = ?, A = ?, B = ?

Applications:

1. Distinguish between a complete quadratic and an incomplete quadratic. Examples are given.
   a. $4X^2 - 7X + 6 = 0$ complete
   b. $3X^2 - 6 = 0$ incomplete
   c. $2X^2 - X = 0$ incomplete

d. $8A - 6 = 2A^2$ complete

e. $-8 = 2A^2$

f. $-4C = 3C^2$

2. Which part of the equation is being factored so as to replace the "?" marks?

   a. $3X^2 + 13X + 4 = (3X + ?)(X + ?)$

      ? = Factors of ______________

   b. $12X^2 + 13X + 1 = (?X + 1)(?X + 1)$

      ? = Factors of _______________

Summary:

1. What is a quadratic equation?
2. Which parts of the equation are factored?

Homework:

1. Identify the quadratic equation

   a. $X^2 + 5X = 0$

   b. $X + 5 = 0$

   c. $-8 + 2X = 3X$

   d. $X^2 + 5X = 24$

2. Which equations are complete (A) or incomplete (B)

   a. $X^2 + 3X = 0$

   b. $-3X^2 - 5 = 0$

   c. $-2X + 4 = -5X^2$

   d. $X^2 = 4$

TOPIC 53: Quadratic Equations - II

Aim: How do we solve quadratic equations?

Performance Objectives: Students will be able to:

- explain how to write a quadratic equation in standard form.
- state the zero product property.

Vocabulary: quadratic equation, standard form, zero product property

Materials:

Chalkboard, chalk
Hardware: Apple IIe computers (16); pair off students
Software: Intellectual software - Quadratics Equations II (L20)
Use of the Apple lab for this lesson - hands-on!

Motivation:

1. Factor
   - a $X^2 + 5X + 6$
   - b. $X^2 - 4X + 4$
   - c. $X^2 - 11X + 24$
2. Solve for X:
   - a. $X + 9 = 0$
   - b. $X - 2 = 0$
   - c. $X(-2) = 0$

Development:

1. Discuss motivation. Elicit: When two numbers add up to zero, they are opposites (additive inverses). Ask the students to name pairs of numbers whose sum is zero. Ask the students to name pairs of numbers whose product is zero. Elicit: When the product of two numbers is zero, at least one of the numbers must be zero. Using logical notation, we may write (AB = 0)---->(A=0 V B=0). Is the converse true? (yes) Therefore the antecedent and consequent are equivalent statements and we may write a biconditional: (AB=0)<--->(A=0 V B=0). A zero product is obtained if and only if at least one factor is zero.

2. Use of software by the students in the computer lab - choose menu option L20.

3. Multiple choice questions follow immediate feedback!

4. Five examples are given for the students to rewrite, factor and then solve.

5. An appropriate set of examples is presented so that the students learn to:

   a. transpose terms to one side

   b. factor completely

   c. set each factor equal to zero and solve for 2 solutions.

6. An appropriate set of application problems is then presented.

Applications:

A well-developed set of review exercises are given on the menu (L25) review/practice.

Summary:

1. What is a quadratic equation?

2. What procedure is used to find the solution set of a quadratic equation?

3. Why should you transform quadratic equations into standard form?

4. Will every quadratic equation have two different roots? (No: Consider $x^2 - 8x + 16 = 0$.)

Homework:

1. Which of the following are quadratic equations?

   a) $x^2 + 1 = x^3$

   b) $r^2 - 1 = 0$

   c) $s^2 + s = 6$

   d) $y - 9 = 0$

   e) $1 + x = x^2$

   f) $u^2 + \frac{1}{u} = 5$

2. Compute: 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 x 0.

3. Solve:

   a) $(x-1)(x-2) = 0$

   b) $(y-7)\ (y-11) = 0$

   c) $(u + 1)(u - 3) = 0$

   d) $(2r - 1)(3r - 1) = 0$

   e) $(s - 1)(s - 2)(s - 3) = 0$

   f) $x(x + 2) = 0$

   g) $p(p + 1) = 0$

   h) $5(2c - 3)\ (3c + 2) = 0$

4. Factor; then solve:

a) $x^2 - 3x + 2 = 0$

b) $p^2 - 16 = 0$

c) $2u^2 - 18 = 0$

d) $v^2 - 2v + 1 = 0$

e) $2r^2 + 3r + 1 = 0$

f) $5s^2 + 10s = 0$

g) $10t = 2t^2$

h) $x(x + 2) + 1 = 0$

TOPIC 54: Quadratic Equations - III

Aim How do we solve a quadratic equation?

Performance Objectives: Students will be able to:

- state the recommended procedure for solving quadratic equations.
- solve quadratic equations.
- check the solution.

Materials

Software: Solving Quadratic Equations (Microcomputer Workshops Courseware)
Hardware: Apple II, IIe, or IIc Computers, Monitor, and Disk Drive

Rationale for Computer Use:

This software presents quadratic equations in the standard form. It provides instruction in solving these equations. The computer allows the students to work at their own pace giving them constant reinforcement each step of the way. In solving the equations, if the correct factors are not entered, an error message is printed with the product of the incorrect factors. The students can see why the factors are not correct and are given second chances. If the students still do not respond correctly; a tutorial section explains how to factor quadratic expressions.

Motivation

Which equation is easier to solve? Explain.

1. $x(x + 2) = 0$ 2. $x(x + 2) = 24$

Development:

1. Discuss motivation. Elicit that the first equation is easier to solve because it already shows a product equal to zero. Students should recall from the previous lesson that this implies that one factor must be zero, $x = 0$ or $x + 2 = 0$ (i.e., $x = -2$).

2. Let's use the computer to review the procedure for solving quadratic equations.

3. Insert the Solving Quadratic Equations disk and turn on the monitor and computer.

4. Choose the instructions option on the main menu.

5. Notice that all of the quadratic equations will be presented in the standard form.

6. How many options are you given in solving these equations? (3; divide both sides by a number, factor the left side, and set each factor equal to 0 and solve)

7. After you have factored each expression correctly, what option must you choose? (set each factor equal to 0)

8. Do a sample example for practice before you begin to work on examples on your own.

9. Follow the computer's instructions as you proceed through the sample example.

10. If you want you can review the instructions for solving the example or you can proceed to the actual examples.

11. Do 4 examples. (2 easy and 2 more difficult)

12. After you complete each example examine your total error analysis to see where you might have made mistakes.

13. Exit the program and turn off the computer when you are finished.

Summary:

1. What are the steps used to solve a quadratic equation?

2. Find the error in the following:

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x - 3 = 0 \qquad x = 3$

$x - 2 = 0 \qquad x = 2$

Check

$x^2 - 5x + 6 = 0$

$3^2 - 5(2) + 6 \overset{?}{=} 0?$

$9 - 10 + 6 \overset{?}{=} 0$

$-1 + 6 \overset{?}{=} 0$

$+5 \overset{?}{=} 0$ (false)

Homework:

1. What conclusion may be drawn from each of the following (a solution is not called for):

a) $xy = 0$

b) $xy > 0$

c) $xy < 0$

d) $(2r + r - 3)(5r - r - 4) = 0$

e) $x^2 < 0$

f) $x^2 + y^2 = 0$

2. Solve and check:

   a) $(x^2 - 9)(2x - 1) = 0$

   b) $(y^2 + y - 6)(3y + 1) = 0$

   c) $(\frac{4}{u} - 2)(u + 1) = 0$

   d) $r^2 = r + 6$

   e) $s(s + 2) = 15$

   f) $2w^2 - w + 3 = 0$

   g) $7p^2 + 2p - 5 = 0$

   h) $5 = 3p + 8p^2$

   i) $2x^2 - x = 15$

   j) $v^2 - v = 42$

   k) $w^2 + 2w = 35$

   l) $6x^2 - x - 12 = 0$

3. Find two consecutive odd integers whose product is 63.

4. A certain rectangle is three feet longer than it is wide. If its area is 54 sq. ft., what is its perimeter?

5. A certain number increased by five times its reciprocal is 6. What could this number be?

TOPIC 55: Circumference of a Circle

Aim: How do we find the circumference of a circle?

Performance Objectives: Students will be able to:

. state what is meant by the circumference of a circle.
. state the formulas for circumference.
. find the circumference of a circle, given the radius or diameter.
. find the radius given the circumference.

Vocabulary: circumference, radius, diameter

Materials:

Any Lotus 1-2-3 clone (VIP Professional used in this case) will follow the outlined keystrokes exactly. Any electronic spreadsheet for any computer can be used provided you read the documentation. Tape measures or string and ruler are needed as well.

Rexo described below also required. Seven different circular objects should be given to each group of students using a computer. Paper circles will suffice but more concrete objects are preferable.

Teacher Preparation:

Teacher must have sufficient copies of the spreadsheet about to be developed on separate data disks, depending on the number of computers to be used. To set up the spreadsheet, follow the instructions carefully. First load the spreadsheet and insert the data disk according to the documentation accompanying the software. Using an 80 column screen, change the column-width globally to 15 by typing /WGC15 and pressing RETURN. Then right justify all label prefixes (or column headings) by typing /WLR and pressing RETURN. Enter the column headings (or titles): CIRCUMF., DIAMTR., and CIRCUMF/DIAM by using the Cursor and RETURN keys to move from cell to cell. In cell C3, type the formula +A3/B3. Then copy this formula into the next 6 cells by typing /C. When asked range you are copying from, either press RETURN (if you are still at C3) or type C3 and press RETURN. When asked range you are copying to, type C4..C9 and press RETURN. Move Cursor to C11 and type @AVG(C3..C9) and press RETURN to designate the range of values you wish averaged. Then label cell B1 "AVERAGE= and press RETURN. Protect entire worksheet from student errors by typing /WGPE. Then unprotect the cells you wish students to have access to typing /RU and when asked for the range, type A3..B9 which will allow students to enter data into the circumference and diameter columns. Save sufficient numbers of this worksheet on seperate data disks using the supplied documentation.

On the chalkboard, or on rexo, set up a table that translates sixteenths of an inch into decimal equivalents as follows: 1/16=.0625,
2/16=.125, 3/16=.1875, 4/16=.25, 5/16=.3125, 6/16=.375,
7/16=.4375, 8/16=.5, 9/16=.5625, 10/16=.625, 11/16=.6875,
12/16=.75, 13/16=.8125, 14/16=.875, 15/16=.9375.

Also prepare a rexograph of the attached "history of pi" to be given to the class at the beginning of the period.

Motivation:

Load the appropriate worksheet into each of the computers to be used, preferably before the period starts. Students are to copy conversion chart at the beginning of period. Tape measures (inexpensive paper tape measures are available from Association of Teachers of Mathematics usually for free or just postage) or pieces of string and rulers are distributed at beginning of period as well.

Instruct students to read the "history of pi" rexo. Remind students that mathematicians have been trying to decipher the mysteries of pi for better than two thousand years, and have tried to derive pi accurately to more and more decimal places during all these years.

Question- Does anyone remember what the uses of pi are?
Elicit- measuring circles for area and circumference

Question- Why would better approximations for pi be useful?
Elicit- You can obtain more accurate measurements of circular or round objects.

Question- Why were the early approximations of pi less accurate than those of today?
Elicit- human error in calculations that today's computers avoid, human error in measuring objects visually, lack of standardized measuring devices that are truly accurate.

Tell students that we are going to see how good an approximation for pi we as a class can come up with using a desktop computer, and a simple measuring device such as a tape measure. "Let's see if a team consisting of 32 students and 16 microcomputers can do better than the ancient mathematicians did with string, papyrus, stylus and mental calculation."

Using containers with rectangular (cardboard box) and circular (waste basket) tops and a tape measure (or string and ruler, if tape measures are not available), the teacher should demonstrate how to measure the length of the edge of the opening. Elicit that for the rectangle this measurement is called the perimeter. Announce that for the circle the measurement is called the circumference.

The teacher should then demonstrate how to use the tape (or string) to measure the diameter of the circle (longest chord). The class should be divided into groups, and each group given one or more circular objects (hula hoop, coffee cup, oatmeal box, etc.): measure the circumference and diameter of the circle.

Development:

Instruct students to tabulate the results and enter them into the first two columns of the spreadsheet. The resulting ratio or quotient will appear instantly in the third column. Remind the students that if any results seem way out of line, they should redo the measurements for that object. The bottom value on the spreadsheet keeps track of the average as each new piece of data is entered. On most spreadsheets that average, will not appear until all 7 objects' measurements have been entered. Then, collect all the final averages and average them either with the spreadsheet or a calculator in order to get the final result for the class. Throw out the two upper and lower averages to see how that affects the class' overall average approximation of pi. If the students were careful, it should work out pretty close.

Note:

The spreadsheet itself will generate pi to a good number of decimal places by typing @PI in an empty unprotected cell.
Continue with the rest of section 1 of the development.

IN THIS LESSON, THE COMPUTER IS BEING USED TO SPEED UP AND MAKE MORE ACCURATE THE STUDENTS' APPROXIMATION OF PI USING THE CIRCUMFERENCE TO DIAMETER RATIO. IF YOU WISH THE STUDENTS TO GAIN A BETTER UNDERSTANDING OF THE HUMAN ERROR FACTOR, HAVE THEM TRY THE CALCULATIONS BY HAND AS PART OF THEIR HW ASSIGNMENT.

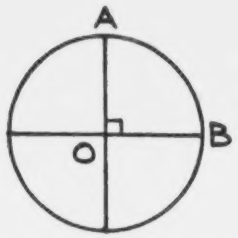
1. Tabulate the results. Students should observe that in every case the circumference is slightltу more than three times the diameter. Divide to find the $\frac{C}{D}$ ratio in each case. The teacher should explain that the differences are due to approximations caused by the rough measurement methods used. If perfectly precise measurements were made, the $\frac{C}{D}$ ratio would be exactly the same in every case. This constant is an irrational number (Review: a nonterminating, nonrepeating decimal), approximately equal to 3.1415926536. Computers have been used to calculate thousands of decimal places of this number. The name commonly used to refer to the number is "pi", the name of the Greek letter . This letter was used for the sound we represent by "P", and was chosen to represent this number to emphasize the connection with the "perimeter" of any circle. A rational number close to the value of π is $3\frac{1}{7}$ ($\frac{22}{7}$), which has the decimal value $3.142857\overline{142857}$. $\frac{22}{7}$ and 3.14 are commonly used as approximations of the value of π.

2. From $\frac{C}{D} = \pi$ obtain $C = \pi d$. Develop: Since $d = 2r$, $C = 2\pi r$. This formula may be used to find the length of the circumference when the radius is known.

3. Exercises such as the following should be presented. (Answers may be approximated or left in terms of π.)
   a) If the radius of a circle is 4, find the circumference.
   b) If $d = 10$, find C.
   c) If $C = 36\pi$, find r and d.
   d) If r is tripled, how does C change?

$$C_1 = 2\pi r$$

$$C_2 = 2\pi(3r) = 6\pi r$$



4. Find the length of (minor) arc AB if OB = 6 where O is the center of the circle.

5. A wheel is rotated three times along a road without slipping. If the diameter equals 2 ft., find the distance the center of the wheel travels. How many times must it rotate to travel 100 ft?

Summary:

1. What does circumference mean?

2. What is the relationship between the diameter of a circle and its circumference?

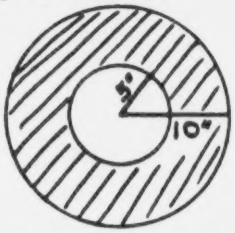
3. What is the relationship between the radius of a circle and its circumference?

4. What is π?

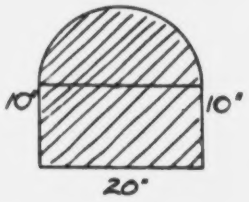
Homework:

1. For each of the following, compute the perimeter of the shaded region in terms of π. Using 3.14 for π, estimate to the nearest whole number your answer.

a)

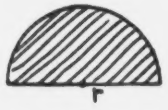


b)

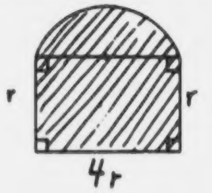


2. In terms of π and r, express the perimeter of the shaded region.

a)

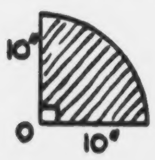


b)

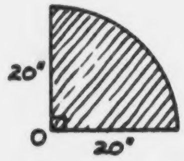


3. Compute the perimeter of the shaded region to the nearest whole number using 3.14 for π.

a)



b)



How do your answers in a) and b) compare?

4. Do example 3 using a 60 degree angle at 0 instead of a 90 degree angle.

5. The circumference of a circle is 1000m. Find to the nearest meter its radius. (Use 3.14 for π.)

6. The perimeter of a semicircle is 1000m. Find to the nearest meter its radius. (Use 3.14 for π.)

THE HISTORY OF ATTEMPTS TO APPROXIMATE PI

In approximately 250 B.C., the Greek mathematician Archimedes showed that pi lies somewhere between 233/71 and 220/70 using circumscribed and inscribed polygons of increased numbers of sides.

In approximately 40 B.C., the Roman architect Vitruvius gave 3-1/8 as his approximation for pi (note: Archimedes did better two centuries earlier.

In the late 1500's, the French mathematician Vieta used Archimedes' methods but increased accuracy by using polygons having as many as 393,216 sides to 10 decimal places.

In the early 1600's, the English mathematician, Brouncker used a method known as continued fractions to get pi to 10 decimal places.

In 1768, the German mathematician Lambert proved pi was irrational, therefore pi has no final decimal place. Future efforts would only get more and more decimal places but no end.

In the 1800's, Shanks, an English mathematician, spent his entire life trying to get more decimal places. He used an unending series of expressions, each gotten from the one before to reach an astounding 707 decimal places for pi by 1873. Unfortunately, in 1944, it was discovered that he had made a mistake at the 528th decimal place so that everything that followed (years of work) was wrong.

Since then, computers have been used to calculate pi to over 100,000 decimal places. More powerful computers will someday be used to improve on that still further.

TOPIC 56: Coordinate Geometry - Graphing Linear Equations in Two Variables

Aim: How do we graph linear equations in two variables?

Performance Objectives: Students will be able to:

- evaluate open sentences in two variables to determine whether given ordered pairs are elements of the solution set. (Determine if a point lies on the line.)
- determine three ordered pairs in the solution set of a given linear equation.
- use the coordinate plane to draw the graph of the line that represents all solutions to a given linear equation.
- state why the equation is called linear.

Vocabulary: collinear, noncollinear, replacement set, linear equation

Materials:

Graph paper, graph stencil for chalkboard, yardstick (optional)
Hardware: 16 MS DOS computers
Software: Algebra Plus; publisher: Microglyph - 1984

Motivation:

1. Solve the equations:

   a. $2X + 5 = -3$
   b. $Y = X + 7$

Development:

1. Discuss motivation problems. The first equation is a review: an open sentence in one variable (usually) yields one element in the solution set. Have two students at each of the computers and have them take turns. They should choose #2 on the menu - "equations in one variable". Students will get immediate feedback and a step by step analysis of each problem after they complete it. The student may exit this part of the program at any time. The program has an option for the second problem in the motivation $Y = X + 7$. It will describe when an equation has an infinite number of ordered pairs. If the replacement set is the set of real numbers the solution set will be infinite.

2. There is a game that allows for finding points, and then coordinates are found that are in the solution set for a specific equation. A table is filled in by the student after a range for X is given. The students pick the X values randomly from within the range and then solve for the Y. The student gets immediate feedback for the choices for X and their answers for Y.

   The use of this program will greatly enhance this lesson since the students can work at their own pace and the partners or pairs can share ideas. Challenge the students to find a point that will not lie on the line. The students will then return to their seat to formalize the procedures developed.

3. Consider the equation $2x + y = 7$. (Have students copy the following procedure as it is elicitied from the class.)

Procedure to Graph a Linear Equation:

1. Solve for Y in terms of X.

   Result: $Y = -2X + 7$

2. Find at least three ordered pairs in solution set by choosing values for X. (Keep X values small, choose 1 positive, 1 negative, and 0.)

   Result: (-2, ?)
   (0, ?)
   (3, ?)

3. Calculate the Y for each chosen X-value.

   Result: (-2, 11), (0, 7)
   (3, 1)

4. Plot points represented by the ordered pairs.

5. Use a straight edge to draw the line through the collinear points. If the points are not collinear, check the work.

6. Label the line with the orginal equation.

7. Check all labels: X-axis, Y-axis, origin, axes numbered to indicate scale line. Ask, "Why are three points located before this line is drawn?" (Two points are sufficient to determine the location of the line; the third point is for "insurance". If they do not appear collinear, recheck your calculations to determine which point(s) is misplaced.)

Applications:

1. Tell whether the given ordered pair is a solution of the open sentence:

   a. $2X + Y = 11$ (3,5)

   b. $X - Y > 11$ (7,-8)

   c. $X + Y = 6$ (-3,-3)

   d. $4X + 3Y = 16$ (4,0)

2. Use the procedure above to graph $X - Y = 8$.

Summary:

1. How would you determine whether a given point lies on a line? (See if the X and Y values of its location satisfy the equation of the line.)

2. Give examples of equations which produce straight lines when graphed.

3. Describe the procedure you would employ to produce the graph of the solution set for a linear equation.

Homework:

1. a. Find 3 points whose coordinates satisfy the equation Y = X + 1, and draw the line.

   b. Find a point that lies on the line just drawn. Do the coordinates of this point satisfy the equation Y = X + 1?

2. a. Find 3 points whose coordinates satisfy the equation X + Y = 10.

   b. Draw the line containing these points .

   c. Find a point on this line whose coordinates are not integers. Check whether its coordinates satisfy X + Y = 10.

3. For the equation 2X + 3Y = 11, find the corresponding values of X or Y that satisfy this equation in the table:

X		Y
1	:	?
?	:	1
0	:	?
?	:	0
	:	

   a. Plot and draw a straight line through these points.

TOPIC 57: Coordinate Geometry - Algebraic Method, Simultaneous Linear Equations

Aim: How do we use algebraic methods to determine the solution set of a system of simultaneous linear equations.

Performance Objectives: Students will be able to:

- solve systems of linear equations in two variables by the use of the addition method
- check the solution by replacing variables in the original equations

Vocabulary: Method of elimination by addition

Material:

Hardware: Apple II
Software: Elementary Algebra: Systems of Equations by Control Data Publishing, Objective 3a

Motivation:

Calling upon previous lessons on graphing, note the inconvenience of constructing a graph even on a computer. Note disadvantages of using a computer to solve systems of equations by graphing.

Development:

1. To solve systems of equation through the use of addition call upon the principles of addition as follows:

   a. if $a = b$ and $c = d$ then $a + c = b + d$

   Place the following problem on the board.

   b. $2x - 3y = 8$ $\qquad$ $-4x + 3y = -4$

   Discuss the solution by adding up like terms of each equation.

2. At this point the student can clearly follow along by turning to their computers and selecting object 3A of the computer software.

3. As a check, teacher asks student to name the terms involved in producing the sums of the 2 equations.

4. What were the sums of these 2 equations and what were the values of x and y? Could we graph this?

5. As a follow-up if time allows students could use "Graph Master" to check all algebraic solutions generated from the specific problems in the Plato Software.

Summary:

1. Today we learned to use the method of elimination by addition for a system of linear equations. How does this method work? (If X or Y variables have the same coefficient but opposite signs, we can use the addition property of equations to eliminate that variable.)

2. What are the advantages of using the algebraic method of elimination by addition?

3. Why is it important to verify that the solution-pair satisfies both equations?

Homework:

1. Solve each system of equations algebraically and check your solution

   a. $-3X + Y = 4$
      $3X + 4Y = 1$

   b. $1.5X + 3Y = -3$
      $-1.5X + 2Y = 5$

   c. $X + 2.5Y = 2$
      $-2X - 2.5Y = 1$

   d. $1.5X + \frac{1}{3Y} = -1$
      $-1.5X + \frac{2}{3Y} = 7$

2. **Challenge Exercises:**

   a. Solve graphically and algebraically:

      $Y = 2X - 5$
      $Y = 2X + 7$

   b. Explain each result and explain how they confirm each other.

TOPIC 58: Coordinate Geometry - More Difficult Simultaneous Linear Equations

Aim: How do we solve more difficult systems of simultaneous linear equations by algebraic methods?

Performance Objectives: Students will be able to:

- manipulate the coefficients of a system of two linear equations so that one of the variables may be eliminated by the addition method.
- confirm graphical solution by algebraic methods.
- check solutions to systems of linear equations.

Vocabulary: constant term, equivalent equation, elimination
review: multiplicative property of equations

Materials:

Chalkboard, chalk
Hardware: One large monitor with an Apple IIe computer
16 Apple IIe computers
Software: Plato - Elementary Algebra: System of Equations. Control Data Publishing 1983. Student should have a paper and pencil

Motivation:

1. Have the students take their chairs and sit on the side of the room. Using the chalkboard ask them to solve:

   a. $X + 4Y = 12$
   b. $2X - Y = -3$

2. Ask the question - "How is this set of linear equations different from the ones we worked with yesterday?"

3. What happens if we add them together?

Development:

1. What was our aim yesterday? If students cannot recall or if they show any lack of understanding have them watch the demonstration on the computer again.

2. Have a student try to discover what to do using the chalkboard. Elicit from the students that we need one variable with the same coefficient or equivalent equations.

3. Have several students complete the solution of the set of equations and formalize on the board what must be done to solve such a set of equations. The students can go to the computers and solve equations of both types and work at their own pace with the help of Microman. (In the software it is illustrated that adding the equation without changing the coefficients, if this is needed, will result in one equation with two unknowns).

Applications:

1. The students using the computer will get sets of 3 problems to solve and get immediate results with immediate feedback for every set of three.
2. The Management system is excellent. The students should have to solve, using pencil and paper, any problems which they fail to solve using the computer.
3. Sets of 2 students at a machine should work together to get a solution to the problem.
4. Teams according to rows can gain points after each set of 3 problems.

Summary:

Elicit the procedure for solving a system of simultaneous linear equations by the addition method:

1. Transform each equation so that variables appear in one member and a constant is in the other member.
2. Use the multiplicative property of equality to situation in which the coefficient of a variable is the additive inverse of the coefficient of the same variable in the second equation.
3. Eliminate one of the variables by the method of elimination by addition.
4. Find the solution set and check.

Homework:

1. In each case, solve for x and y using addition and the multiplicative properties of equations. Check their common solution in both equations.
   - a) $2x + y = -1$
     $x - 2y = -3$
   - b) $3x + 2y = -7$
     $-x + 2y = -3$
   - c) $x + 1.57 = 0$
     $x - 3y = -9$
   - d) $3x - 2y = -10$
     $4x - y = -10$
2. Solve: $\frac{x}{2} + \frac{y}{3} = 3$, and check.

$$\frac{x}{3} + \frac{y}{2} = \frac{1}{3}$$

3. **Challenge Exercise**:
   Use a system of linear equations to solve:
   Three shirts and two ties cost $53.00 while two shirts and three ties cost $47.00. What is the cost of one shirt and one tie?

TOPIC 59: Coordinate Geometry - Simultaneous Linear Equations - Game Situation

Aim: How do we show mastery of solving simultaneous equations using a game situation and the microcomputer?

Performance Objectives: Students will be able to:

- solve any set of linear equation using the techniques for the solution of
- simultaneous equations, discovered in previous lessons in a game setting using the computer.
- check the answers of any set of equations in which an error occurs before going on to the next problem.

Materials:

Chalkboard, colored chalk
Hardware: 16 Apple IIe computers
Software: Plato - Elementary Algebra: System of Equations. Control Data Publishing 1983.

Motivation:

1. The teacher should be totally familiar with the diagnostic and mastery test disk of this set of software.

2. The teacher should go through the entire set of problems before the class is given this as a game/competition. If the student has not mastered the easier problems, Example:

   a. $2X - 3Y = 8$
      $X + 3Y = 4$

   b. Another set should be presented before going on to the more difficult problem types:

      $2X + 3Y = 1$ or $3X + 5Y = 8$
      $-1X - 2Y = 2$ $\quad$ $2X + 3Y = 4$

   c. The teacher can set this up as a contest/game situation in which the pairs of students work on the problems as a mini-team that is part of a larger team. Each mastery test score is tallied and the winning team for each set receives a point. At the end of the class a small reward can be given to each member of the winning team.

Development:

1. (See above) Microman takes part by keeping score during each mastery test. A point is recorded only if the problem is solved correctly.

2. Each student uses pencil and paper to solve each problem and he/she gets immediate feedback through the excellent management system built into this software. This approach is excellent since the teacher is free to walk around the room and help absentees from the previous days and also to discover error patterns. The computer is a motivational device in

itself and feedback from the students about the software is valuable also. This software is User Friendly.

3. The teacher just inserts the software and the students are ready to go. It is a good idea to have the software numbered and to collect it before the end of the class to insure that none is removed from the room.

Summary:

The student will discover where errors are being made when they check their solutions. This will heighten the importance of checking the answers. The software does not stress checking but the teacher often must facilitate the gaining of certain techniques and not totally rely on the software to teach all the skills we want the students to gain. The software should be used to enhance and clarify key skills.

Homework:

1. Solve graphically and algebraically

   a. $2X - Y = 5$

      $X + Y = 4$

   b. $2X - 5Y = 8$

      $-X + Y = 2$

TOPIC 60: Coordinate Geometry - Algebraic Solution of Linear Equations - Substitute Method

Aim: How do we solve a system of linear equations algebraically by the method of substitution?

Performance Objectives: Students will be able to:

- solve systems of linear equations in two variables by the method of substitution.
- check the solution by replacing variables in the original equations

Vocabulary: Method of substitution, absolute value

Motivational Question:

How could we solve the following system of equations without the use of addition?

$$2x + y = 3$$
$$x - 2y = 4$$

Materials:

Hardware: Apple II
Software: Elementary Algebra: Systems of Equations by Control Data Publishing, Objective 2A

Development:

1. Have students turn to computer and ask what method does (software 2A) its program tell us to use?
2. What does the computer tell us to do with the value of x?
3. Represent x in terms of y.
4. What does the computer suggest we do with this value? etc.
5. Continue with objective to the next problem.
6. Now have students solve $-x + y = 4$; $x + 2y = 2$ utilizing the substitution method (follow this along with computer)

Summary:

1. Name the three methods we have learned for finding the solution set of a system of simultaneous linear equations.
2. Tell which method you would select for each of the following systems. Explain your choices.

$$\begin{array}{lll} y = 5x - 3 & 2x + 3y = 4 & 2x - 3y = 8 \\ y = -2x + 11 & -3x + y = -17 & y = 2x + 4 \end{array}$$

What other factors would you consider in selecting a method?

Homework:

1. Solve using the method of substitution. Then check your solution in both original equations.

   a) $3a + 2b = 20$
   $b = 2a - 11$

   b) $r = 3s + 5$
   $2r + 3s = -1$

   c) $6u + v = 20$
   $4u + 5v = 22$

2. Solve by using two different methods. Check your solution in both equations.

   a) $5x + 2y = -3$
   $x - y = -2$

   b) $2x = 4y + 1$
   $4x - 2y = 8$

   c) $1.5x + y = 1$
   $2.5x + 3y = 11$

3. The smaller of two numbers is 6 less than twice the greater. If their difference is 1, find the two numbers.

4. Challenge Exercise:
   A father is 3 times as old as his son. In twelve years, the father will be twice as old as his son. What are their present ages? Let x represent the father's age and y represent the son's.

   $x = 3y$

   twelve years hence: $x + 12 = 2(y + 12)$

Software Duplication Guidelines

Like plays, novels, songs, textbooks, and all other published material, the programs and files on a published computer diskette are fully protected by the copyright laws of the United States. No diskette can legally be copied without the appropriate permission of the publisher. The position of the Division of Computer Information Services of the New York City Board of Education on diskette copying is the following:

1. Duplication of copyrighted software or documentation is prohibited.
2. Software publishers must be contacted to verify procedure for back-up multiple copies and licensing agreements.
3. A publisher's written permission is required to download or network programs to other microcomputers.
4. No student or Board of Education employee may use school computers to duplicate copyrighted software.
5. The use of illegally duplicated software, however obtained, is prohibited in all Board of Education facilities.
6. Simultaneous use of original software and/or its back-up is not permitted without the written permission of the publisher.

The copyright of the enclosed diskettes, however, belongs exclusively to the New York City Board of Education. Any New York City public school employee may make an unlimited number of copies of these diskettes for use in the New York City public schools, either in day or evening sessions, without obtaining further permission. For your convenience, and to enhance the educational value of these programs, it is strongly advised that you make at least one copy of each diskette for each computer in your computer lab to allow all of your students to work on the same program simultaneously.

The New York City Board of Education has computer diskettes under contract at this time. If you wish to purchase additional diskettes, the Division of Computer Information Services suggests that you contact:

Jordan Computer Enterprises Inc.
1436 Gordon Street
Allentown, Pa. 17102
contract number P27291
vendor number COM121

Copying Public Domain Diskettes for MS DOS Computers

Diskcopy with One Disk Drive

To make a copy of a software diskette on an IBM PC, PCjr, or IBM compatible computer with a single disk drive, follow these steps:

1. Before you start, be sure that the diskette you wish to copy is write-protected. Write protecting a diskette will prevent you from accidentally erasing or overwriting any of its programs or files. You can write-protect a diskette by placing a small piece of tape over the write-protect notch on its left side.

2. Insert the IBM DOS diskette into the disk drive, turn the computer and the monitor on, wait for the DOS to boot up, then enter the date and time. The last line on the screen will be A>.

3. Type Diskcopy, then press ENTER. The screen will display the message,

   ```
   Insert SOURCE Diskette in Drive A :
   Press any key when ready . . .
   ```

   The SOURCE diskette is the diskette you want to copy. Remove the DOS diskette from the disk drive and insert the Source diskette. Press ENTER.

4. The screen will display a Copying . . . message, then the following instructions:

   ```
   Insert TARGET Diskette in Drive A :
   Press any key when ready . . .
   ```

   The TARGET diskette is the diskette you want to copy to. Remove the source diskette from the disk drive and insert the Target diskette.

   (NOTE: It is not necessary to format a new diskette before using it as the target diskette. Diskcopy will format automatically before copying.)

   Press ENTER.

5. If your target diskette has not been formatted before, the screen will display a Formatting . . . message before it copies. After it has formatted and copied, it will repeat the message it gave you in step 4:

   ```
   Insert SOURCE Diskette in Drive A :
   Press any key when ready . . .
   ```

   Remove the target diskette from the disk drive and insert the source diskette again. Press ENTER.

6. After another Copying . . . message, the screen will repeat the message from step 5:

   ```
   Insert TARGET Diskette in Drive A :
   Press any key when ready . . .
   ```

Again, remove the source diskette from the disk drive and insert the target diskette. Press ENTER.

7. This is the last stage of the procedure. When the screen displays the message

   ```
   Copy another diskette (Y/N)?
   ```

   the procedure is over; the target diskette has been made into a duplicate of the source diskette.

8. If you want to make another copy of the source diskette, type in Y. The screen will repeat the message

   ```
   Insert SOURCE Diskette in Drive A :
   Press any key when ready . . .
   ```

   You can repeat this procedure as many times as necessary and make as many copies of your original diskettes as you need for use in the New York City public schools. When you no longer wish to make additional copies, press N and remove the source diskette from drive A.

Diskcopy with Two Disk Drives

Copying on an IBM PC or IBM-compatible computer with two disk drives is an even easier procedure, because it doesn't require alternately removing and inserting the source and target diskettes. Follow the directions for steps 1 and 2 for a computer with one disk drive, then type in Diskcopy A: B: at step 3 and press ENTER. The screen will display this message:

```
Insert SOURCE Diskette in Drive A :
Insert TARGET Diskette in Drive B :
Press any key when ready . . .
```

Remove the DOS diskette, insert the source and target diskettes into the proper disk drives and press any key on the typewriter keyboard. The screen will display a Copying . . . message while it copies the source diskette and a formatting . . . message if the target diskette hasn't already been formatted. When the screen displays the message

```
Copy another diskette (Y/N)?
```

the procedure is over; the target diskette has been made into a duplicate of the source diskette. If you want to make another copy of the source diskette, type in Y and repeat the procedure.

Copying Public Domain Diskettes for the Apple II

To make a copy of your software diskette, follow these steps:

1. Before proceeding, be sure that the diskette you wish to copy is write protected. Write protecting your diskette will prevent you from accidentally erasing or overwriting any of its programs or files during this or any other use of the diskette. You can write protect a diskette by placing a small piece of tape over the write protect notch on the diskette's left side.

2. Insert the DOS 3.3 System Master diskette into drive 1 and turn the computer and the monitor on. When the Applesoft prompt and cursor appear appear type in RUNCOPYA and press RETURN.

3. The screen will display the message

```
APPLE DISKETTE DUPLICATION PROGRAM
ORIGINAL SLOT:   DEFAULT = 6
```

Press RETURN.

4. The screen will display

```
ORIGINAL SLOT: 6
        DRIVE:    DEFAULT = 1
```

Again, press RETURN.

5. The screen will display

```
       DRIVE: 1
```

and add a new message:

```
DUPLICATE SLOT:  DEFAULT = 6
```

Press RETURN.

6. The screen will display

```
DUPLICATE SLOT: 6
```

and ask one last question:

```
       DRIVE:    DEFAULT = 2
```

If you are using an Apple II Plus or IIe with one disk drive, change the number 2 to 1 and press RETURN. If your machine has two disk drives, simply press RETURN. The screen will display

```
-- Press 'RETURN' key to Begin Copy --
```

7A. If you are using a computer with two disk drives, the screen will give you no further instructions. Remove the DOS 3.3 System Master diskette and insert the source diskette into drive 1. (The source diskette is the diskette you want to copy.) Insert the target diskette into drive 2.

(NOTE: It is not necessary to format a new diskette before using it as the target diskette. The Apple II Plus or IIe will format it automatically before copying onto it.)

Press RETURN. Reading and Writing messages will appear alternately as the computer reads the source diskette and writes onto the target diskette. If the target diskette has not been formatted before, the screen will also display Formatting.

7B. If you are using a computer with a single disk drive, press the RETURN key. You will have to alternately insert and remove both the source diskette and the target diskette during the copying procedure. The screen will tell you what to do at the proper times. Follow its instructions carefully. When you insert the target diskette for the first time, the screen will display the word Initializing while it proceeds.

When the screen displays the message

```
DO YOU WISH TO MAKE ANOTHER COPY?
```

the procedure is over; the target diskette has been made into a duplicate of the source diskette.

8. If you want to make another copy of the source diskette, type in Y and press RETURN. The screen will repeat the message

```
-- Press 'RETURN' key to Begin Copy --
```

You can repeat this procedure as many times as necessary and make as many copies of your original diskettes as you need for use in the New York City Public Schools. When you no longer wish to make additional copies, press N and remove the source diskette from drive A.

Caring for Your Diskettes

Your computer diskettes will provide years of dependable service if you and your students handle them properly by following a few simple procedures.

1. Never handle a diskette unnecessarily. When it is necessary to touch a diskette, hold it by the corner near the label, and touch as little of its surface as possible. Under no circumstances should you ever touch the interior magnetic recording surface, either through the center of the diskette or any other opening in the diskette cover.

2. Never bend or fold a diskette.

3. Whenever possible, write out the diskette label before affixing it to the diskette. If it is necessary to write on the label after it has been affixed to the diskette, always use a felt tipped pen or marker. Never write on the diskette with a pencil or ball point pen.

4. Never attach anything to a diskette with a paper clip of any kind.

5. When a diskette is not in use, always return it to its paper envelope. store it in a cool, safe place, away from direct sunlight, excessive heat, moisture, dust, and dirt of any kind.

6. Computer diskettes are magnetic, and any contact with other sources of magnetism can destroy the programs and files that are stored on them. Keep your diskettes away from magnets of any kind, no matter how harmless they seem to be. Magnetic desktop paper clip holders (a principal source of trouble because they aren't obvious), magnetic children's toys and so forth can damage your diskettes.

   In addition, many common electric and electronic appliances are sources of magnetism. Televisions, radios, electric lights, and even the screen of your computer monitor produce minute amounts of magnetism which can cause diskette damage. Store your diskettes away from these devices when they are not in use, and do not allow your diskettes to come into contact with them when you are using them.

7. Never attempt to clean a diskette with cloth or fluid.

8. Always write protect your important diskettes by covering the notch in the left side with tape. This will keep you from accidentally erasing or overwriting an existing program or file.

9. Finally, accidents that ruin or erase diskettes are bound to happen. Learn how to use the diskcopy and copy commands on your DOS, and, as a precaution, always make back-up copies of any diskettes you would find difficult or impossible to replace.

For More Help...

The Division of Computer Information Services maintains a Technical Assistance Center (TAC) in each of the five boroughs. These centers are equipped with a wide variety of computers, software, and, most important, experienced advisors to help you use the computers in your school more effectively and to answer any questions about computer education you may have. All New York City Public School teachers are welcome to make use of TAC facilities, either by telephone or in person, at any time during the year. TACs are normally open from 9 A.M. to 5 P.M. during the school year and through all school vacations, including the summer, but daily hours can vary. Call first.

Bronx TAC
1000 Teller Avenue
Bronx, N.Y. 10456
(212) 588-0279

Brooklyn TAC
3000 Avenue X
Brooklyn, N.Y. 11235
(718) 769-2860

Manhattan TAC
134 West 122 Street
New York, N.Y. 10027
(212) 662-0122

Queens TAC
167-01 Gothic Drive
Jamaica, N.Y. 11432
(718) 658-0491

Staten Island TAC
1200 Manor Road
Staten Island, N.Y. 10314
(718) 494-5042

List of Software Used in This Guide

Algebra Plus	Microglyph	MS DOS	90-3510.13.8
Appleworks	Apple	Apple	61.8160.03.X
Basic Math Competency - Graphs: Mean, Median and Mode	Educational Activities	Apple	89-5258.41.2
Basic Math Competency - Graphs: Mean, Median and Mode	Educational Activities	MS DOS	89-5258.41.2
Binomial Multiplication	Microcomputer Workshops	Apple	60-4510.38.1
Binomial Multiplication	Microcomputer Workshops	MS DOS	90-4510.09.3
Elementary Algebra: Polynomials	Control Data Publishing	Apple	60-3510.44.2
Elementary Algebra: Exponents, Roots and Radicals	Control Data Publishing	Apple	60-3510.30.2
Elementary Algebra: Polynomials	Control Data Publishing	Apple	60-3510.44.2
Elementary Algebra: Systems of Equations	Control Data Publishing	Apple	60-3510.34.5
Elementary Logic - Level I	Courses by Computer	Apple	60-3511.64.3
Equations	Microcomputer Workshops	Apple	60-3511.20.1
Equations II	Microcomputer Workshops	Apple	60-3511.52.X
Equations III	Microcomputer Workshops	Apple	60-4510.35.7
Factoring Algebraic Expressions	Microcomputer Workshops	Apple	60-4510.30.6
Geometric Presupposer	Sunburst Activities	Apple	91-1831.34.7
Geometry Alive	Educational Activities	Apple	60-3510.81.7
Geometry Alive	Educational Activities		
Integers (A Contest)			
Introduction to Geometry	Soc F/Vis Ed	Apple	89-2902.44.2
Lotus 1-2-3	Lotus	MS DOS	
Practical Algebra Series	Intellectual Software	Apple	60-4510.35.7
Quadratic Equations II	Intellectual Software	Apple	87.2009.94.7
Rocky's Boots - Lab Ed. (6 disks)	Learning Co.	Apple	91-7995.75.9
Rocky's Boots - School Ed. (2 disks)	Learning Co.	Apple	91-7995.46.5
Rocky's Boots - Lab Ed. (6 disks)	Learning Co.	Commodore	91-7995.93.3
Rocky's Boots - School Ed. (2 disks)	Learning Co.	Commodore	91-7995.86.4
Rocky's Boots - School Ed. (2 disks)	Learning Co.	IBM	91-7995.55.4
Rocky's Boots 25-1139	Learning Co.	MS DOS clone	55-5082.17.2
Scott Foresman Geometry - Quadrilaterals	Scott Foresman	Apple	67-3149.45.5
Solving Quadratic Equations	Intellectual Software	Apple	60-4510.40.3
Special Topics in Mathematics	Intellectual Software	Apple	60-2510.52.X